

1. Advanced linear algebra (Chapters 9 and 10)

- Vector spaces, linear dependence and independence, basis.
- Inner products, Cauchy–Schwarz inequality, triangle inequality Gram–Schmidt method.
- Matrix of a linear operator, change of basis.
- Systems of linear equations, Gaussian elimination, rank, inverse matrix.
- Determinants and their properties.
- Eigenvalues and eigenvectors, diagonalization, diagonalization of self-adjoint matrices by unitary transformations, quadratic forms.
- Applications: principal moments of inertia, first-order linear systems of ode’s (exponential of a matrix), normal modes of vibration, uncertainty principle.

2. ODEs (Chapters 11 and 12)

- Linear equations with constant coefficients.
- Inhomogeneous equations.
- General first- and second-order equations.
- Power series method.
- Classification of singular points.
- Method of Frobenius at a regular singular point.
- Method of obtaining a second solution — example of Bessel function of second kind.

3. Sturm–Liouville theory (Chapters 14 and 15)

- Eigenvalue problem for differential equations: self-adjoint Sturm–Liouville equations — role of BC’s.
- Self-adjoint Sturm–Liouville equations have real eigenvalues, orthogonal and complete eigenfunctions.
- Eigenfunction expansions: Parseval’s relation (equality), Bessel’s inequality, Riemann–Lebesgue lemma. Formula for coefficients, weighted inner products.
- Applications of eigenfunction expansions: orthogonal polynomials, Bessel eigenfunctions, Fourier series, Fourier sine and cosine series (odd and even extension of a function), Gibb’s phenomenon, convergence rate.

4. PDEs (Chapter 16)

- Wave equation in 1-dimension: modelling of vibrating string, d’Alembert’s solution.
- Separation of variables solution of second-order linear PDEs, for example:
 - Vibrating rectangular membrane.
 - Vibrating circular membrane.
 - Laplace equation in spherical coordinates.

The exam

The references to chapters of the textbook are meant only to help you study; they do *not* define the topics for the exam. The class notes define the course material. The exam contains 4 questions. Questions will be similar to those appearing on the assignments. A significant portion of the exam relates to material covered after Test 2.