Assignment 2 **MATH 3C** **26 September 2005**

1. Apply the Gram-Schmidt procedure to the functions

$$u_1(x) = i, \quad u_2(x) = ix, \quad u_3(x) = ix^2,$$

(where $i = \sqrt{-1}$) to produce a set of functions $v_1(x), v_2(x), v_3(x)$ which is orthonormal with respect to the inner product

$$\langle f, g \rangle = \int_0^\infty f^*(x) g(x)\, e^{-x}\, \mathrm{d}x.$$

Use the fact that $\int_0^\infty x^k e^{-x}\, \mathrm{d}x = k!$ for $k = 0, 1, 2, \ldots$ (where $0! = 1$).

2. Let $\{1,\ x,\ x^2,\ x^3\}$ (c any real number) be a basis (call it the u basis) for V, the vector space of polynomials of degree ≤ 3, with inner product $\langle f, g \rangle = \int_{-1}^{1} f(x)g(x)dx$.

   (a) Apply Gram-Schmidt to construct an orthogonal basis $\{v_1, v_2, v_3, v_4\}$ (do not normalize).

   (b) Find the matrix C which converts u to v.

   (c) Find the matrix D_v of the linear operator d/dx with respect to the basis v.

   (d) Calculate $C^{-1} D_v C$. What is this matrix?

3. (a) Find the determinant of the following matrix:

$$\begin{vmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{vmatrix}.$$

   (Recall that adding a multiple of one row or column to another does not change the determinant.)

   (b) Find det(T) for the linear operator on R^3 defined by $T(x, y, z) = (2x - z, x + 2y - 4z, 3x - 3y + z)$. (Hint: first find the matrix of T with respect to, e.g., the standard basis.)

4. Consider the matrix

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}.$$

   (a) Find the characteristic polynomial of A.

   (b) Find the eigenvalues of A.

   (c) Find bases for all eigenspaces of A.

5. Text: §10.2 problem 8 (p. 470).

6. Text: §10.3 problems 15, 16, 17, 18 (p. 480). These are really all the same problem!