Assignment 1 **MATH 3C** **12 September 2005**

1. Consider the complex vector space P (the set of all polynomials with complex coefficients) and the subsets M of all those vectors (polynomials) p for which

   (a) p has degree 3,

   (b) $2p(0) = p(1)$,

   (c) $p(t) \geq 0$ whenever $0 \leq t \leq 1$,

   (d) $p(t) = p(1-t)$ for all t.

   In which of these cases is M a subspace of P?

2. (a) Show that the three vectors $(1, 1+i, 1)$, $(0, i, 1)$, $(1, i, 0)$ are linearly independent.

   (b) Do they form a basis for C^3?

   (c) Use row reduction with complex numbers to express the vector $(2-2i, 1-i, 1+i)$ as a linear combination of the three vectors in 3(a).

3. Text: §9.5 Problem 18 (p. 443).

4. Consider the first four Legendre polynomials $P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2-1), P_3(x) = \frac{1}{2}(5x^3 - 3x)$.

   (a) Show that (P_0, P_1, P_2, P_3) are linearly independent, and that they form a basis for the vector space of cubic polynomials.

   (b) Now consider the operator $\mathcal{A}$ defined by

   $$\mathcal{A}p = -p + 2\frac{dp}{dx}.$$

   Show that $\mathcal{A}$ is a linear operator.

   (c) Construct the matrix representation of $\mathcal{A}$ with respect the basis (P_0, P_1, P_2, P_3).

   (d) Apply the linear operator $\mathcal{A}$ to the vector $u = a + bx + cx^2 + dx^3$ using the matrix you found in (c).

5. Text: §9.7 Problem 6 (p. 453).

6. (a) Given functions $a_j(x)$ $(j = 1, \ldots, n)$, consider the homogeneous linear ordinary differential equation

   $$\sum_{j=0}^{n} a_j(x)y^{(j)} = a_n(x)y^{(n)} + \cdots + a_2(x)y'' + a_1(x)y' + a_0(x)y = 0.$$

   Show, in detail, that the solutions to this equation form a subspace (of the vector space of continuous functions).

   (b) Do the solutions of a homogeneous nonlinear equation, such as

   $$y'' + yy' + 3y = 0,$$

   form a subspace? Explain.