SPEAKERS, TITLES, AND ABSTRACTS
GEOMETRY AND TOPOLOGY OF MANIFOLDS
MCMASTER UNIVERSITY
MAY 14–18, 2004

Speaker. Selman Akbulut, Michigan State University
Title. *4-manifold corks and fillings*
Abstract. We will discuss "corks" and "palfs" on 4-manifolds (they are developed jointly with S.Matvayev and B.Ozbagci, respectively) Every smooth 4-manifold M can be decomposed into 2 codimension zero Stein submanifolds, put together along their common boundaries, $M = N \cup W$, and W can be taken to be contractible (cork). This decomposition enjoys certain uniqueness property (Matveyev's thesis). Furthermore, any 4-dim Stein manifold X can be made a certain Lefschetz fibration P over 2-disk (palf). Existence of palf first proven by Loi-Peirgallini, and later Ozbagci and I gave a canonical construction from the Stein handlebody. Viewing a palf P as a primitive of a Stein manifold $X = |P|$ has many advantages: For example it gives an open book on the boundary, and gives a certain canonical compactification of the corresponding Stein manifold X (more generally "symplectic manifold with boundary) into closed symplectic manifolds by two steps: (1) attaching a 2-handle to the binding (Eliashberg recently observed that this operation can be made symplectic by making the symplectic form exact at the attaching region), (2) extending across the open book by monodromies (this operation is already symplectic, as was previously shown with Ozbagci). Finally, we will discuss how to construct exotic smooth manifolds by using the action of corks on Ozsvath-Szabo's Heegard Floer homology (recent joint work with S.Durusoy).

Speaker. Denis Auroux, Massachusetts Institute of Technology
Title. *Near symplectic structures and Lefschetz pencils on smooth 4-manifolds*
Abstract. Every compact smooth 4-manifold with nonzero b_+ carries a near-symplectic structure, i.e. a closed 2-form which degenerates in a specific way along a union of circles. The study of these structures is largely motivated by Taubes' program to interprete Seiberg-Witten invariants in terms of enumerative geometry in this context. In this talk, we will describe an extension to the near-symplectic situation of Donaldson's construction of Lefschetz pencils (this is joint work with S. Donaldson and L. Katzarkov, with most of the credit going to Donaldson). More precisely, we will define the appropriate notion of singular Lefschetz pencil, state the existence result, and discuss some simple topological considerations and examples.

Speaker. Scott Baldridge, Indiana University
Title. *Geography of symplectic 4-manifolds with Kodaira dimension one*
Abstract. The symplectic geography problem can be extended to nonsimply connected manifolds by taking into account the cohomology ring structure of the manifold. In this talk we describe a ring invariant of the symplectic structure, called the degeneracy, which measures how far a symplectic manifold is from satisfying the conclusion of the Hard Lefschetz Theorem. We then show how to fill out the geography of symplectic 4-manifolds with Kodaira dimension one using the usual geography data together with the degeneracy. (This is joint work with Tian-Jun Li.)

Speaker. Igor Belegradek, Georgia Institute of Technology
Title. *Collapsing theory and classification of negatively pinched manifolds with amenable fundamental groups*
Abstract. I will explain how to use the collapsing theory of Cheeger, Gromov, and Fukaya to obtain a diffeomorphism classification of pinched negatively curved manifolds with amenable fundamental groups. (This is a joint work with Vitali Kapovitch.)

Speaker. Steve Boyer, Université du Québec à Montréal
Title. *Characteristic subsurfaces, character varieties and Dehn filling*
Abstract. Let M be a compact, connected, orientable manifold whose boundary is a torus and whose interior admits a complete hyperbolic structure of finite volume. We use character variety arguments and the JSJ theory of characteristic submanifolds to derive near optimal results on the relationship between Dehn fillings of M which yield reducible manifolds and those which yield other sorts of non-hyperbolic manifolds. (This work is joint with Marc Culler, Peter Shalen and Xingru Zhang.)

Speaker. Weimin Chen, Tulane University
Title. *Smooth s-cobordisms of elliptic 3-manifolds*
Abstract. Using Freedman's 4-dimensional topological surgery theory, Cappell and Shaneson, and Kwasik and Schultz completely classified topological s-cobordisms from an elliptic 3-manifold to itself up to orientation-preserving homeomorphisms. In particular, their results showed that there are topologically nontrivial, orientable 4-dimensional s-cobordisms. On the other hand, virtually nothing is known for the smooth category. In particular, it is not known whether there are any exotic smooth s-cobordisms, or whether any of the nontrivial topological s-cobordisms is smoothable. In this talk, I will discuss some ideas of dealing with these questions using methods from symplectic and contact topology. Details can be found in arXiv: `math.GT/0403395` and `math.GT/0403396`.

Speaker. Jim Davis, Indiana University
Title. *Stable diffeomorphisms and a fake connected sum of real projective 4-space*
Abstract. This talk discuss two phenomena in 4-manifold theory. The two topics are unrelated, except in that they are both consequences of high-dimensional manifold theory (aka surgery theory). The first topic is my conjecture that smooth, compact, homotopy equivalence 4-manifolds with torsion-free fundamental group are stably-diffeomorphic, i.e. become diffeomorphic after connected sum with several $S^2 \times S^2$'s. I have proven this for almost every torsion-free fundamental group one can think of, as a consequence of work on the Borel/Novikov conjectures of high-dimensional topology.

The second topic is the recent construction of closed topological 4-manifolds homotopy equivalent to $\mathbb{RP}^4 \# \mathbb{RP}^4$, which are themselves not a connected sum of manifolds with non-trivial fundamental group. These manifolds are smoothable and non-standard after connecting sum with some $S^2 \times S^2$'s, but are standard after passing to a finite irregular cover.

Speaker. John Etnyre, University of Pennsylvania
Title. *Invariants of embeddings via contact geometry*
Abstract. I will describe a method to define, hopefully new, invariants of any embedded submanifold of Euclidean space. To define this invariant we will need to take an excursion into the realm of contact geometry and a recent generalization of Floer homology called contact homology. More specifically, after recalling various notions from contact geometry, I will show how to associate a Lagrangian submanifold to any embedded submanifold of Euclidean space. The invariant of the embedding will be the contact homology of this Lagrangian. Though the definition of this invariant is somewhat complicated it is possible to compute it for knots in Euclidean 3-space. Lenny Ng has combinatorially studied this invariant for such knots and has shown that it does not seem to be determined by previously known invariants but non the less has some connections with the classical Alexander polynomial of a knot. I will concentrate on the more geometric aspects of the invariant and ongoing work of Tobias Ekholm, Michael Sullivan and myself aimed at a better understanding of the invariant (in particular, showing that it is well defined in some generality).

Speakers. Paul Feehan & Tom Leness, Rutgers University & Florida International University
Title. $SO(3)$ *monopoles: The overlap problem*
Abstract. The $SO(3)$ monopole program, initiated by Pidstrigach and Tyurin, aims to compare the Donaldson and Seiberg-Witten invariants by providing a cobordism between the moduli spaces defining these invariants. The main difficulty in this program lies in describing the links of some singularities in this cobordism. Taubesian gluing maps parameterize neighborhoods of subsets of these singularities. The program requires a description of a union of these neighborhoods. We describe some recent work describing the overlap of images of two or more gluing maps.

Speaker. Ron Fintushel, Michigan State University
Title. *Invariants of Lagrangian tori*
Abstract. For nullhomologous Lagrangian tori there is an easily defined framing invariant which can be used to distinguish these tori up to ambient symplectomorphism. In examples constructed from knot surgery this invariant is actually an invariant of diffeomorphism of pairs. We use Seiberg-Witten theory to prove this and give many examples. (This is joint work with Ron Stern.)

Speaker. Kim Frøyshov, University of Bielefeld
Title. *Compactness and non-compactness for monopoles*
Abstract. When one stretches the metric on a compact spin-c 4-manifold X along a hypersurface, the solutions to certain perturbed monopole equations on X exhibit non-compactness phenomena that are unfamiliar from the theory of instantons. After showing examples of this we give sufficient conditions for compactness in the Morse-theoretic sense to hold and discuss consequences for Floer homology.

Speaker. Bob Gompf, University of Texas at Austin
Title. *Stein surfaces as open subsets of* $\mathbb{C}^2$
Abstract. Stein surfaces are open complex surfaces with proper holomorphic embeddings in $\mathbb{C}^n$. A theorem of Eliashberg reduces the construction of Stein surfaces to handlebody theory. We use this technology to investigate the problem of constructing Stein surfaces as open subsets of $\mathbb{C}^2$ and other complex surfaces. There is essentially complete freedom to choose the topological type of such Stein subsets, and a typical topological type results in many diffeomorphism types of Stein surfaces.

Speaker. Ko Honda, University of Southern California
Title. *Controlled constructions of Reeb vector fields*
Abstract. In this joint work with Vincent Colin, I will explain how to construct 'hypertight' contact structures, namely contact structures which have Reeb vector fields without any contractible periodic orbits.

Speaker. Eleny Ionel, University of Wisconsin
Title. *Curves and Gromov-Witten invariants of 3-folds*
Abstract. In this talk we will discuss several conjectures and partial results about the structure of Gromov-Witten invariants of Calabi-Yau 3-folds. These are reminiscent of the situation in 4 real dimensions, where certain Gromov-Witten invariants carry the same information as other gauge theoretical invariants (like Donaldson or Seibert Witten invariants). For a symplectic 3-fold. we will explain the relation between GW invariants and the count of embedded pseudoholomophic curves. The talk is based on joint work with Tom Parker.

Speakers. Stanislav Jabuka & Thomas Mark, Columbia University & Southeastern Louisiana University
Title. *Heegaard Floer homology and mapping tori*
Abstract. We extend and apply techniques developed by Ozsváth and Szabó to give explicit calculations of the Heegaard Floer homology groups of certain 3-dimensional mapping tori. We take the monodromy to be given as a composition of Dehn twists, and consider many of the cases arising when the monodromy is supported on a genus-1 summand of a closed surface of genus $g > 1$. (This is a joint talk with Thomas Mark.)

Speaker. Paul Kirk, Indiana University
Title. *Minimizing the Euler characteristic of 4-manifolds with free abelian fundamental groups*
Abstract. Hausmann and Weinberger defined $c(G)$ for G a finitely presented group to be the minimal Euler characteristic among all closed oriented 4-manifolds M with fundamental group isomorphic to G. They asked what the value $c(G)$ takes when G is a free abelian group of rank n. I'll describe the answer. (Joint work with C. Livingston.)

Speaker. Slava Krushkal, University of Virginia
Title. *Link groups of 4-manifolds*
Abstract. I will introduce a sequence of link groups which are invariants associated to a 4-manifold. In a certain sense they are geometrically dual to the lower central series of the fundamental group, but are more subtle since they are not in general invariant under homotopy equivalences. I will also give an application of these invariants to decompositions of the 4-sphere and in particular to the A-B-slice problem.

Speaker. YiJen Lee, Princeton University
Title. *Seiberg-Witten model for Heegaard Floer homologies*
Abstract. I will introduce a variant of Seiberg-Witten-Floer homologies, which on one hand are equivalent to the ordinary versions via a connected sum formula, on the other hand they are expected to relate to the Heegaard Floer homologies of Ozsvath and Szabo via Taubes's works on SW = Gr.

Speaker. Tian-Jun Li, University of Minnesota
Title. *Symplectic surfaces in symplectic manifolds*
Abstract. We study the existence of embedded symplectic surfaces in a degree 2 homology class of a closed symplectic manifold. We give a complete answer when the dimension of the symplectic manifold is at least 6. For 4-dimensional symplectic manifolds, we describe various constructions and some obstructions.

Speaker. Tomasz Mrowka, Massachusetts Institute of Technology
Title. *From foliations of three manifolds to representations of their fundamental groups*
Abstract. Recent progress in understanding symplectic four-manifolds (due to Eliashberg building on a long line of work in particular that of Gabai , Thurston-Eliashberg and Giroux) combined with results in gauge theory (in particular results of Feehan and Leness relating the Donaldson and Seiberg-Witten invariants) has led to the resolution of some long-standing questions about three manifolds. For example one can now show (independently of Perelman's work) that non trivial surgery on a non-trivial knot can never yield a counter-example to the Poincaré conjecture (the property P question). In general one can show taut foliations of three manifolds imply the existence of non-trivial representations of their fundamental groups in to $SU(2)$.

Speaker. András Némethi, Ohio State University
Title. *The Seiberg-Witten invariant conjecture and projective plane curves*
Abstract. In 2002 L. Nicolaescu and the speaker formulated a very general conjecture which relates the geometric genus of a Gorenstein surface singularity with rational homology sphere link with the Seiberg-Witten invariant (or one of its candidates) of the link. Recently, I Luengo, A. Melle-Hernandez and the speaker found some counterexamples using superisolated singularities. The theory of these hypersurface singularities is equivalent with the theory of cuspidal projective plane curves. In the case when the corresponding curve has only one singular point we were not able to find any counterexample. In fact, in this case the above Seiberg-Witten conjecture led us to a very interesting and deep property of these curves (sitting in algebraic geometry) which seems to generalize famous conjectures and properties (e.g. the Noether-Nagata or the log Bogomolov-Miyaoka-Yau inequalities).

Speakers. Brendan Owens & Sašo Strle, McMaster University
Title. *Definite manifolds bounded by rational homology three-spheres*
Abstract. Given a rational homology three-sphere Y one may ask which definite pairings are intersection forms of manifolds bounded by Y. We discuss various approaches to this problem including the constraints yielded by the Ozsvath-Szabo correction terms of Y. We also discuss resulting bounds on the slice genus of knots and links.

Speaker. Jongil Park, Konkuk University
Title. *New symplectic 4-manifolds with $b_2^+ = 1$*
Abstract. In this talk I will review some known symplectic 4-manifolds with $b_2^+ = 1$ and I will present a new family of simply connected symplectic 4-manifolds with $b_2^+ = 1$ and $c_1^2 = 2$ which are not diffeomorphic to rational surfaces.

Speaker. Jacob Rasmussen, Princeton University
Title. *Khovanov homology and the slice genus*
Abstract. I'll describe some rather striking coincidences between the Khovanov and knot Floer homologies. In particular, I'll explain how the Khovanov homology can be used to define an knot invariant $s(K)$ analogous to the invariant $\tau(K)$ coming from knot Floer homology. Like τ, s gives a lower bound for the slice genus. For certain classes of knots, such as positive knots and alternating knots, it can be shown that $\tau = s$, and (as of right now) it seems possible that $\tau = s$ for all knots. If time permits, I'll discuss some possible explanations for this phenomenon.

Speakers. Danny Ruberman & Nikolai Saveliev, Brandeis University & University of Miami
Title. *Rohlin's invariant and gauge theory*
Abstract. A combination of Casson's and Taubes' work shows that the Rohlin invariant of a homology 3-sphere can be computed by counting flat $SU(2)$ connections. A 4-manifold that is homologically a product of a 3-manifold and a circle has a Rohlin invariant, and we have a series of results that demonstrate a close connection between this Rohlin invariant and Donaldson's gauge-theoretic count of (projectively) flat connections.

In the case of a 4-manifold which is homologically a product of the 3-sphere and a circle, the gauge-theoretic invariant was discussed by Furuta and Ohta. We calculate this invariant for the mapping torus of a finite-order diffeomorphism of a homology sphere via an explicit formula involving the equivariant Casson invariant and a version of the Boyer-Nicas invariant. We deduce that this invariant reduces mod 2 to the Rohlin invariant of a manifold carrying a generator of the third homology group.

We also discuss analogous Rohlin and Donaldson invariants for homology 4-tori, and show that these invariants, suitably normalized, agree mod 2, by showing that they coincide with the quadruple cup product of 1-dimensional cohomology classes. A similar argument works for homology 3-tori to give a purely gauge-theoretic proof that the Casson invariant of a homology sphere reduces mod 2 to the Rohlin invariant.

Speaker. Michael Usher, Massachusetts Institute of Technology
Title. *Lefschetz fibrations and pseudoholomorphic curves*
Abstract. Taking up a program initiated by S. Donaldson and I. Smith, we show how Lefschetz fibration techniques can be used to obtain information about pseudo-holomorphic curves in symplectic four-manifolds. More specifically, we give a new proof of a duality in the Gromov-Taubes invariants which had previously only been known as a consequence of Seiberg-Witten theory, and we prove new vanishing results for certain Gromov-Witten invariants.

Speaker. Stefano Vidussi, Kansas State University
Title. *Isotopy problem for symplectic 4-manifolds*
Abstract. We will discuss various results concerning the problem whether, in a symplectic 4-manifold, two symplectic or lagrangian surfaces that represent the same homology class are isotopic.