

Inferring the spatial scale of environmental variation in sapling recruitment of *Pinus elliottii* (slash pine)

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- Data

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Templates



Templates



Templates



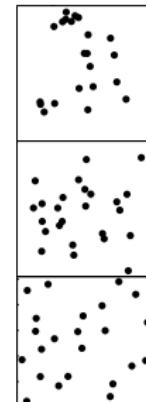
Questions

- How do templates modify spatial distributions in general?
- Can we work out exact expressions for particular cases?
- Can we recover information on templates from messy data?
(How?)

Spatial correlation/covariance

$$c_{ij}(|\mathbf{x} - \mathbf{y}|) \propto \langle (n_i(\mathbf{x}) - \bar{n}_i) \cdot (n_j(\mathbf{y}) - \bar{n}_j) \rangle$$

Positive correlation \iff clustering/association



Zero correlation \iff random (Poisson)

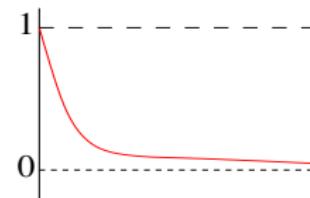
Negative correlation \iff evenness/segregation

Spatial correlations as a function of distance

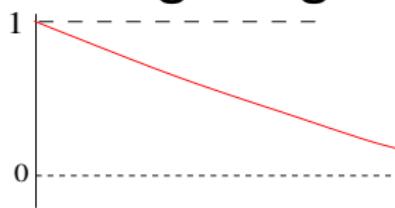
uncorrelated



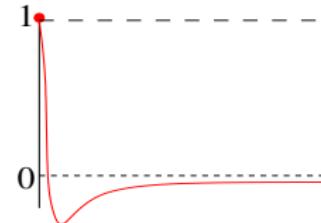
short-range



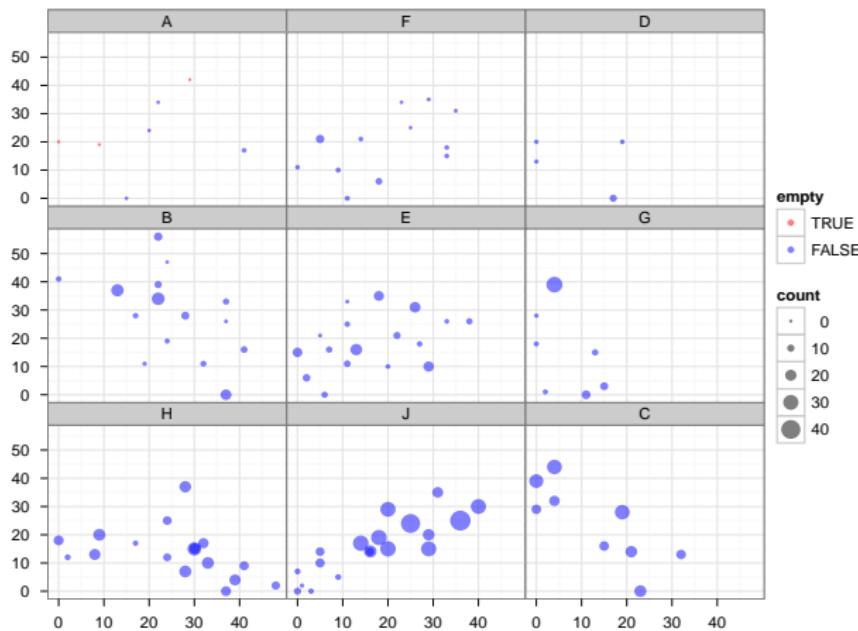
long-range



even

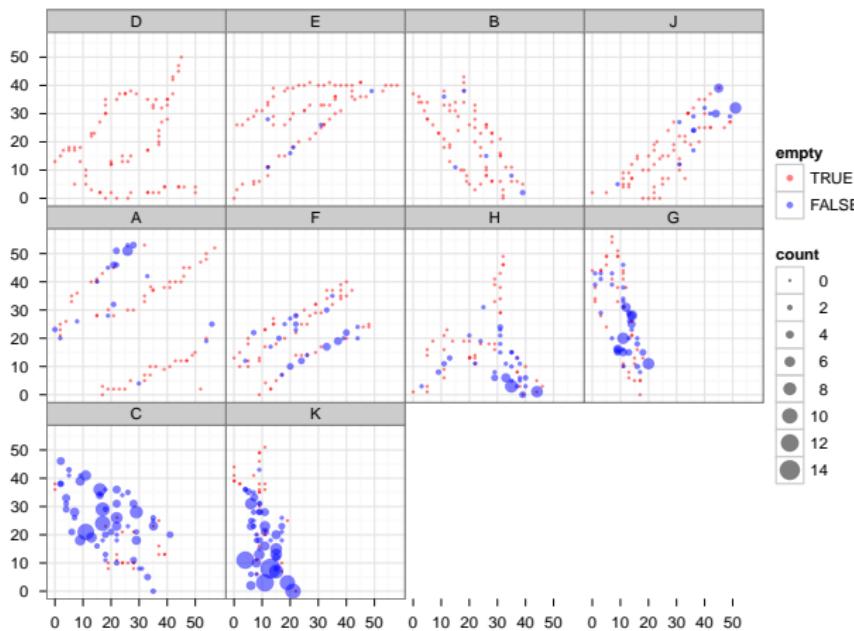


Seed data

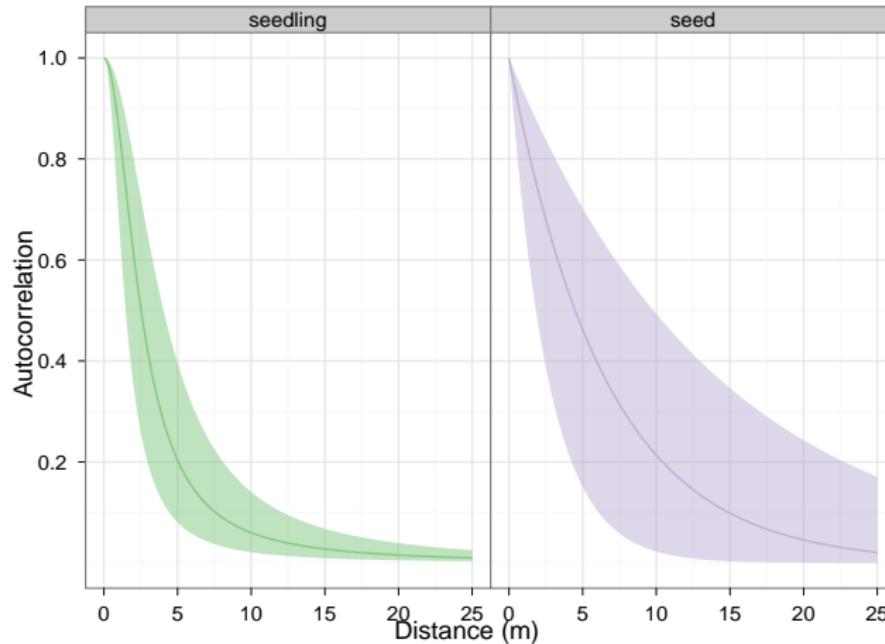




Sapling data



Correlation functions



Analytical framework

- start with a spatial distribution of seeds $N(x) \dots$
- seeds become saplings (recruit) with probability $E(x)$, so $S(x) = N(x)E(x)$ (on average)
- describe spatial pattern (autocovariance) of seeds (C_{NN}); establishment process (environment, C_{EE}); saplings (C_{SS})
- we'll also need means (\bar{N}, \bar{E}), variances ($\text{Var}N, \text{Var}E$)
- **assume** spatial homogeneity/isotropy (!)

Analytical framework (2)

- assume **cross-covariance** $C_{EN} = 0$
no correlation between seeds and environment,
e.g. long-distance dispersal



$$C_{SS}(r) = \bar{N}^2 C_{EE}(r) + \bar{E}^2 C_{NN}(r)$$

(where \bar{N} =mean seed density, , \bar{E} =mean establishment probability)

- or (switch to correlation c)

$$C_{SS} \propto \frac{\text{Var } E}{\bar{E}^2} c_{EE} + \frac{\text{Var } N}{\bar{N}^2} c_{NN}$$

- ... a weighted **mixture** of the two correlation functions

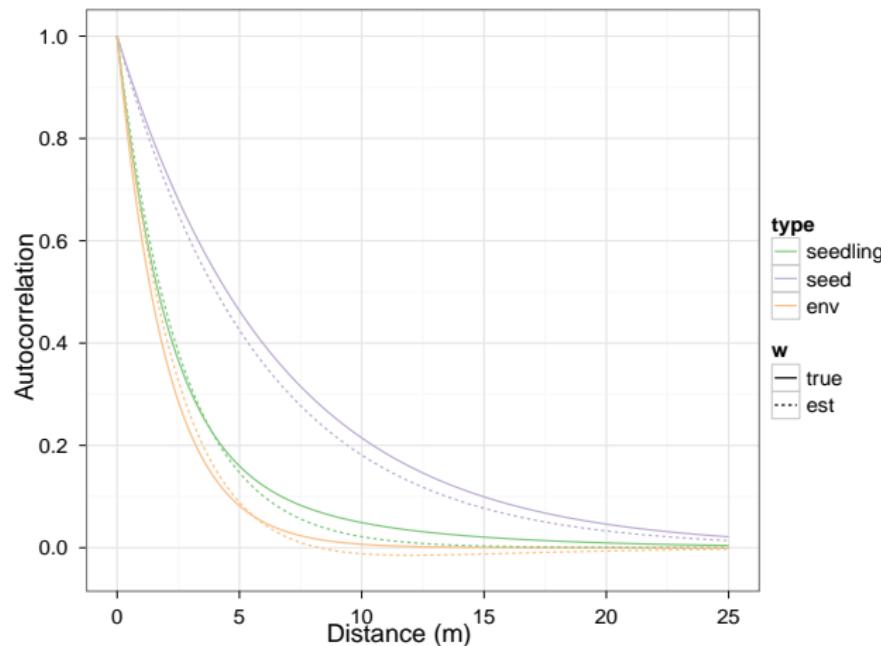
Solving for c_{EE}

Therefore,

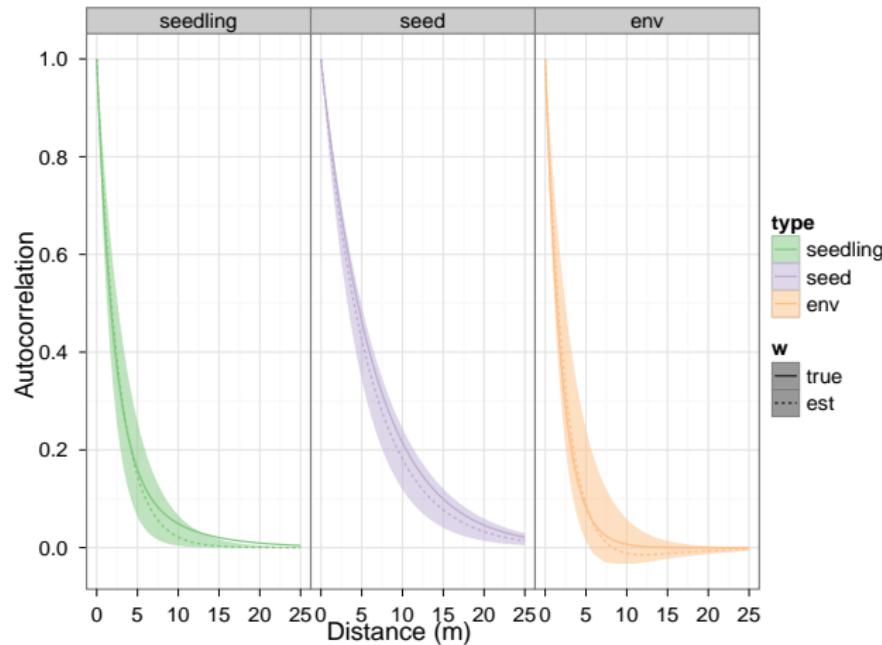
$$c_{EE} \propto \sigma_S^2 c_{SS} - \bar{E}^2 \sigma_N^2 c_{NN}$$

Can we really use this?

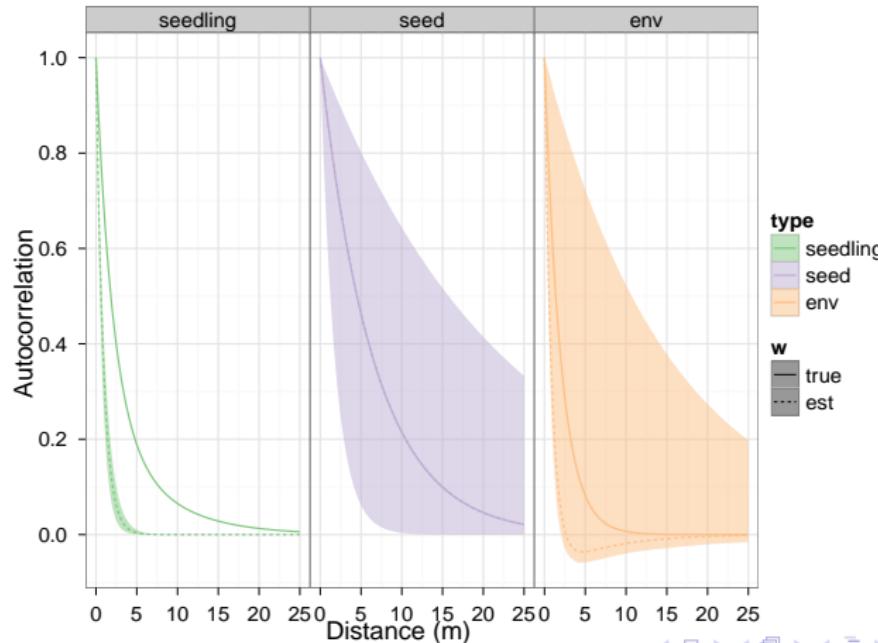
Gaussian simulations

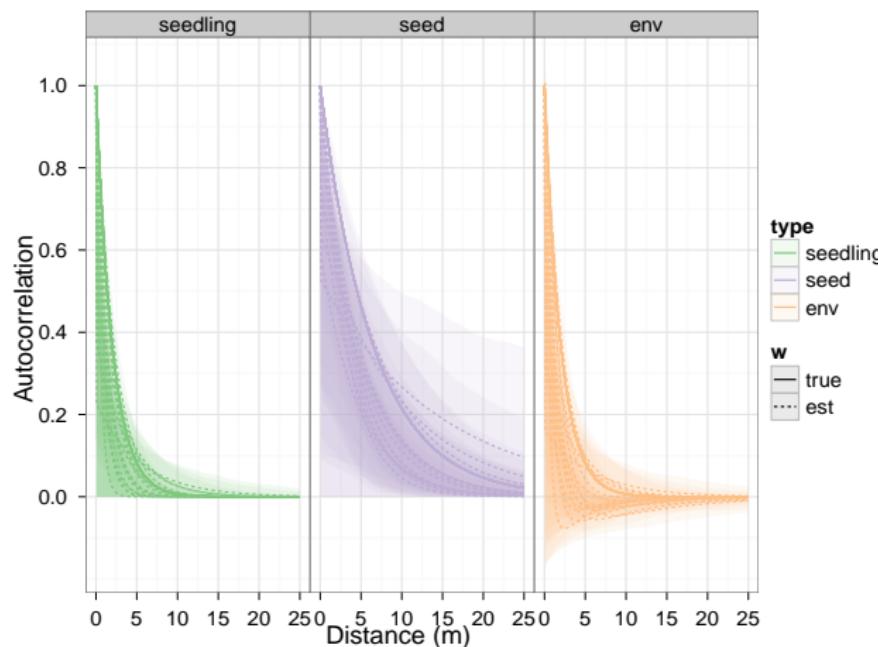


Gaussian simulations (2)

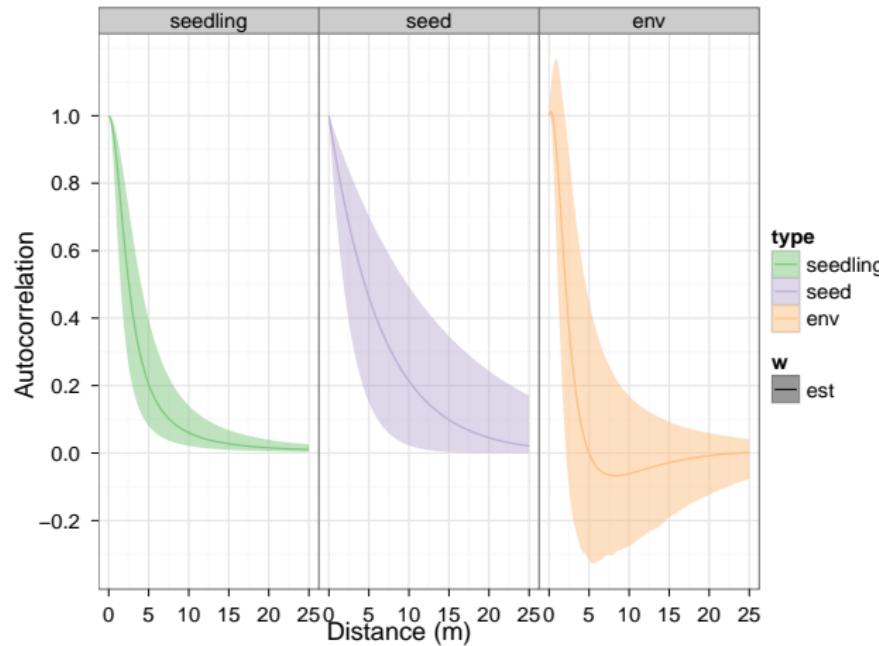


Poisson/binomial simulations





Data (!)



Conclusions

- we **may** be able to recover spatial information
- still need to characterize / understand / correct for bias
- tradeoff between robust pattern analysis and incorporating (sort of) mechanism ?
- technically challenging (used `gls` in R: ask for details!)

Acknowledgements

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