Spatial correlation and deconvolution: (attempting to) estimate spatial process from pattern

Ben Bolker, University of Florida

Case Western University
Biomath seminar

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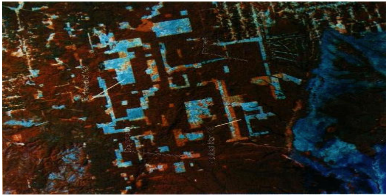
Outline

Ecological landscapes

endogenous/exogenous?

endogenous: foci



anthropogenic patches

fractal?

Interactions in ecological landscapes

Geometries:

- endogenous vs. exogenous
- continuous vs. discontinuous
- disorganized vs. coherent/deterministic

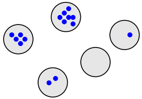
Interactions:

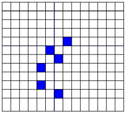
- competition
- local dispersal
- social aggregation
- predator/prey (host/pathogen)
- disturbance

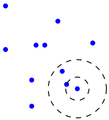
Case study: spatial synchrony

- Many (animal) populations display population fluctuations in *synchrony* across broad geographic regions: grouse, voles, butterflies, lynx and hares . . . e.g. Eastern spruce budworm, *Choristeroneura fumiferna*
- Driving process(es): correlated environmental variation (the *Moran effect*), dispersal, nomadic predators?
- "you can't subtract thunderstorms from ~~lemmings~~ spruce budworms"

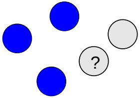
Geometries for spatial ecology

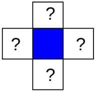


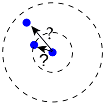




patch/metapopulation **lattice** **point process/IBM**







patch occupancy models ("classical" metapops)

$$p' = cp(1-p) - mp$$

pair approximation

$$p([n0])' = \ldots$$

PDEs *or* **spatial moment equations**

$$\frac{\partial c(r)}{\partial t} = \ldots$$

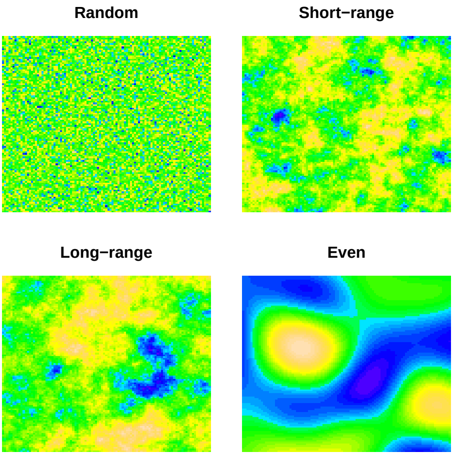
Spatial correlation/covariance

- Describe the relationship between conditions at two points a given distance apart
- Spatial covariance:

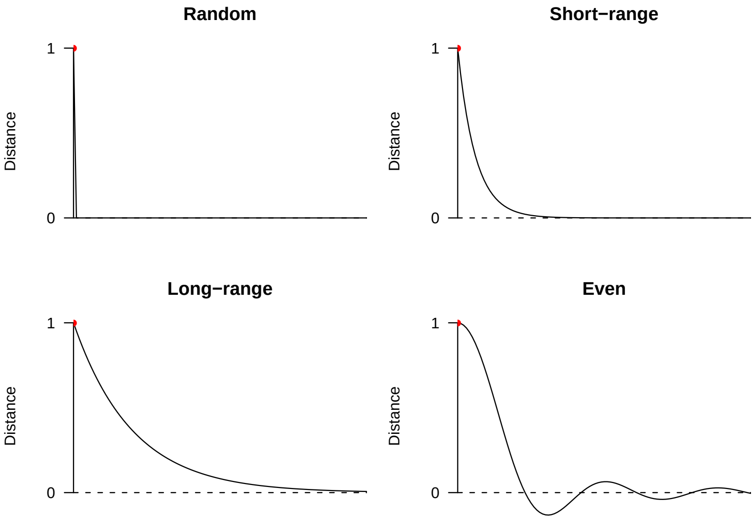
$$c_{ij}(|\mathbf{x}-\mathbf{y}|) \propto \langle (n_i(\mathbf{x})-\bar{n}_i)\cdot(n_j(\mathbf{y})-\bar{n}_j)\rangle$$

- Correlation is a scaled version ($\in [-1,1]$)

Spatial patterns



Spatial correlations as a function of distance



Correlation equations

Define individual-based spatial process
Write down transition rules

Moment equations	**Continuous equations**
Derive expected changes in mean density and spatial correlation	Derive expected changes in density
Assume (second-order) spatial homogeneity	Assume continuous densities
Moment closure	Linearize around equilibrium

Fourier transform correlation equations
Solve for equilibrium correlations/power spectrum

Correlation equations: alternate derivations

via moment equations:

- small-scale patterns
- allows/requires detailed consideration of interactions
- noise terms emerge naturally from dynamics

via continuous equations:

- large-scale patterns
- simple extension of existing models
- allows arbitrary scaling of noise terms

Correlation equations: *via* moment equations

$$\frac{\partial c_{\mu n}(\rho)}{\partial t} = \underbrace{f(D * c_{\mu n})}_{\text{clustering}} - \underbrace{\bar{n} c_{\mu\mu}}_{\text{habitat association}} - \underbrace{\bar{\mu} c_{\mu n} - \alpha \bar{n}(U * c_{\mu n} + c_{\mu n})}_{\text{thinning}}$$

$$\frac{1}{2} \cdot \frac{\partial c_{nn}(\rho)}{\partial t} = \underbrace{f(D * c_{nn} + D\bar{n})}_{\text{clustering}} - \underbrace{\bar{n} c_{\mu n}}_{\text{habitat association}} - \underbrace{\bar{\mu} c_{nn} - \alpha \bar{n}\left(c_{nn} + U * c_{nn} + U\bar{n}\right)}_{\text{thinning}}$$

Correlation equations: *via* continuous eqns

cf. Lande et al. 1999, Engen et al. 2002:

$$\frac{\partial N(\mathbf{x},t)}{\partial t} = \underbrace{F(N(\mathbf{x},t),E(\mathbf{x},t))}_{\text{pop. growth}} - \underbrace{mN(\mathbf{x})}_{\text{emigration}} + \underbrace{m\int D(\mathbf{y},\mathbf{x})N(\mathbf{y})\,d\mathbf{y}}_{\text{immigration}}$$

$$\frac{\partial n}{\partial t} \approx \underbrace{-rn(\mathbf{x},t)}_{\text{regulation}} + \underbrace{m(D*n-n)}_{\text{redistribution}} + \underbrace{\sigma_E^2 e(\mathbf{x},t)}_{\text{noise}}$$

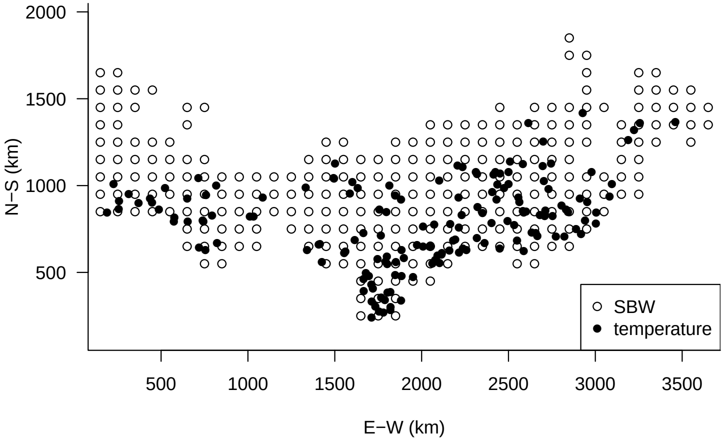
$$2(r+m)c^* = m(D*c^*) + \sigma_E^2 \text{Cor}(e)$$

Combining theory and data

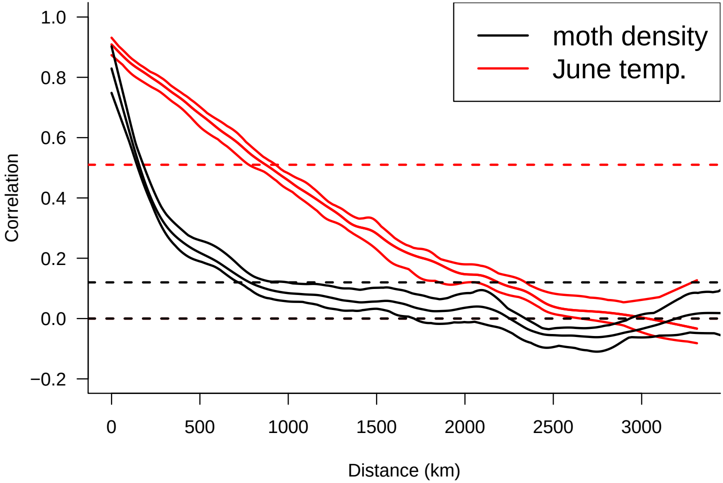
- Classical approach: *empirical* and/or *qualitative* predictions (strong inference)
- Instead, *decompose* spatial patterns by multiple processes: estimation rather than hypothesis testing
- Technical challenge of quantifying spatial patterns and drivers

...

Moth sampling locations



Spatial correlations of moth data



Spatial logistic: solution

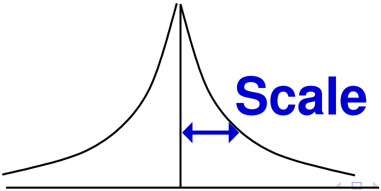
At equilibrium, the power spectrum of the population densities obeys

$$\tilde{S} = \left| \left(\tilde{N}^* \right) \right|^2 = \frac{\sigma_E^2 \tilde{e}}{2(r + m(1 - \tilde{D}))}$$

where ˜ denotes the Fourier transform. Therefore:

$$\text{Scale}^2(p) = \text{Scale}^2(e) + \frac{m}{r}\text{Scale}^2(d)$$

(Lande et al. 1999) where Scale represents the standard deviation of the autocorrelation function *with respect to space*



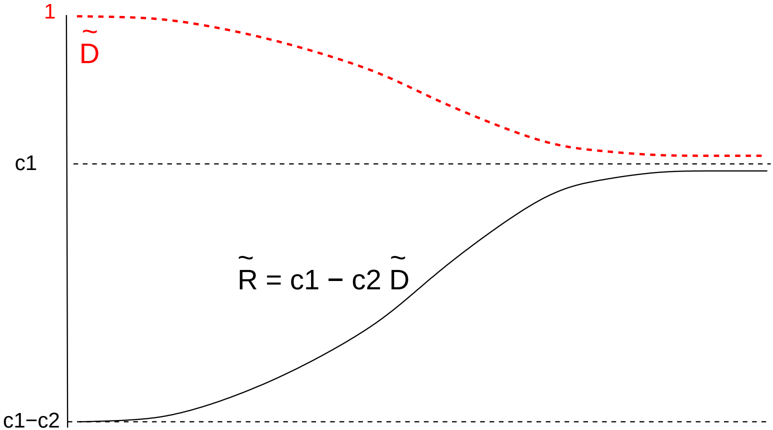
Spectral ratios

Alternatively, we can calculate the *spectral ratio*:

$$\tilde{R} = \frac{\tilde{e}}{\tilde{S}} = \frac{2}{\sigma_E^2}(r + m(1 - \tilde{D})) = c_1 - c_2 \tilde{D}$$

Re-inverting this corresponds to *deconvolving* the effects of environmental variability from the population pattern ...

Reconstructing the dispersal curve



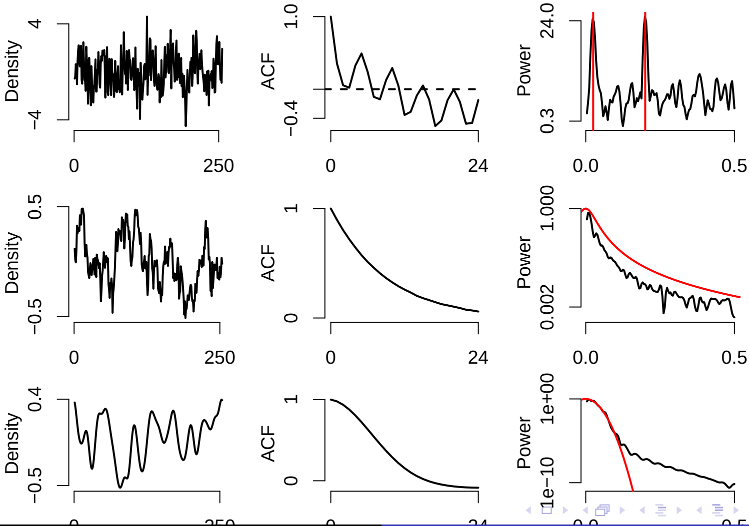
We know the limits $\tilde{D}(0) = 1$, $\tilde{D}(\infty) \to 1$: thus

$$\tilde{D}_{\text{est}}(\omega) = \frac{\tilde{R}(\infty) - \tilde{R}(\omega)}{\tilde{R}(\infty) - \tilde{R}(0)}$$

Spatial power spectra

- Measure variance or *power* accounted for by a given *spatial frequency*
- May be familiar from time series: emphasize different features than correlation analysis (e.g. periodicity)

Landscapes, correlations, and power spectra



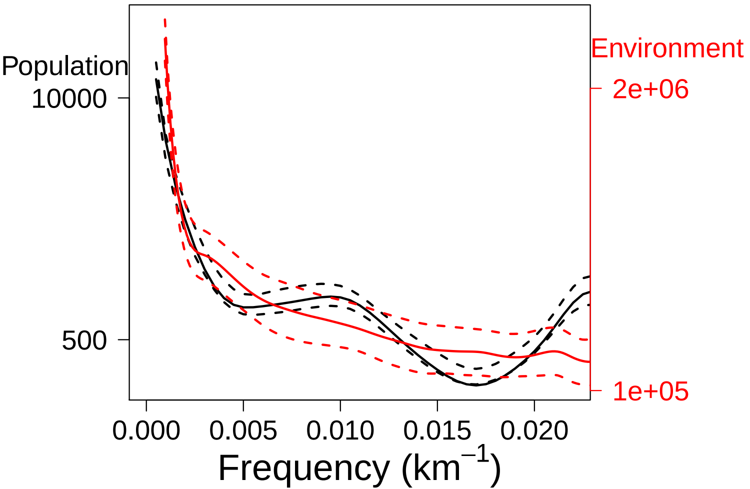
Spatial power spectra: methods

Challenge: unevenly sampled, 2D data

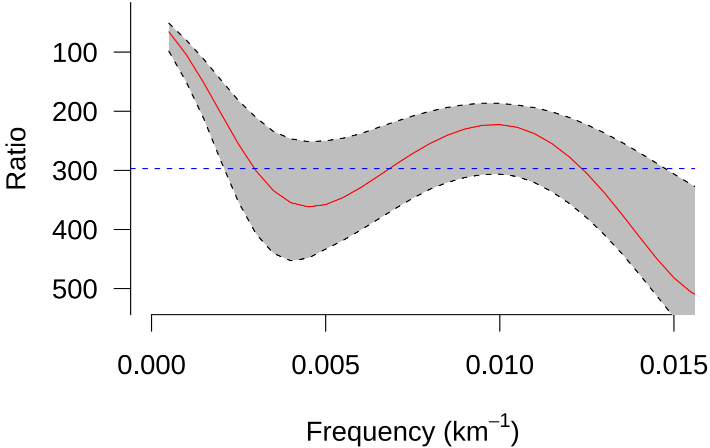
- Nonparametric models (direct Fourier/Hankel transform)
- Geostatistics: simple parametric models (exponential, spherical, etc.)
- Spatial ARMA models

Tradeoff between generality and simplicity/statistical power: biases in nonparametric estimates?

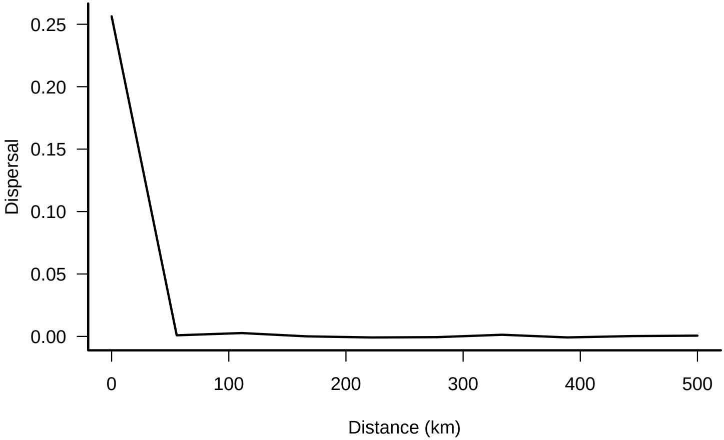
Moth data: spectra



Moth data: spectral ratios



(Putative) moth dispersal curve



- We often want to estimate the effects of some *local* environmental variable (e.g. habitat, competitor density) on an ecological response (population density, fecundity)
- but not sure about the spatial scale of the relationship
- existing methods:
  - regress response on local averages at different scales
  - inverse methods
- what if (1) data are not measured in commensurate ways, (2) we want a nonparametric estimate?

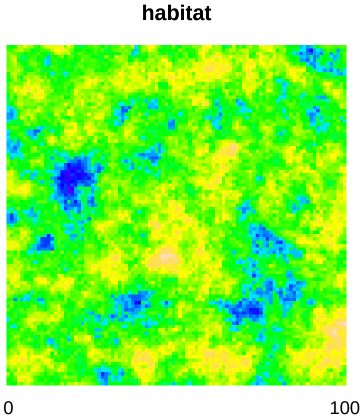
Model

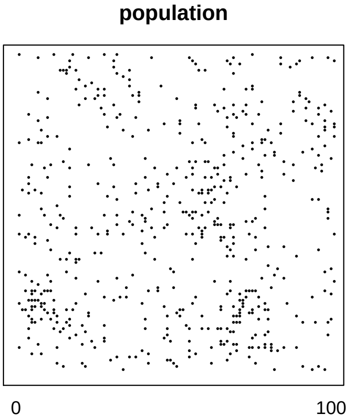
We can think of this as (surprise!) a convolution:

$$N(x) \propto (E * H)(x)$$

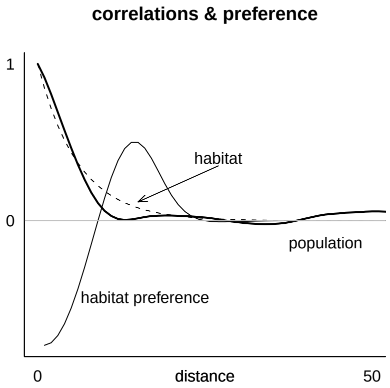
where E is the environmental variable (water?) and H is the *habitat kernel*

example: habitat selection





drivers


correlations & preference
1
0
habitat
population
habitat preference
0
distance
50

Solution

If $N(x) \propto (E * H)(x)$ then

$$C_{EN}(r) = (C_{EE} * H)(r)$$

and ...

$$\tilde{C}_{EN}(\omega) = \tilde{C}_{EE}(\omega)\tilde{H}(\omega)$$

$\tilde{H} = (\tilde{C}_{EN}/\tilde{C}_{EE})(\omega)$ is hence the ratio of the cross-spectrum of (environment $\times$ population) divided by the power spectrum of the environment ...

Caveats, limitations, & challenges

- Fluctuating populations (synchrony vs. population correlation)
- Strongly nonlinear dynamics: *linearization*
- Non-normal responses
- Numerical stability of ratios
- Nonparametric (or flexible) spectral estimates

Correlation equations: applications

- Theoretical analyses of plant competition
- Evolution of dispersal
- Synchrony
- Separating spatial processes in plant competition (esp. step-by-step)

Acknowledgements

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photo credits:
http://www.crh.noaa.gov/mkx/slide-show/tstm/

http://www.pfc.forestry.ca/biotechnology/posters/optical_e.html