Lab 6: estimation (solutions)

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Exercise 1:
(Recreate negative binomial data, likelihood surface, etc.):

```
> set.seed(1001)
> mu.true = 1
> k.true = 0.4
> x = rnbinom(50, mu = mu.true, size = k.true)
> NLLfun1 = function(p, dat = x) {
+     mu = p[1]
+     k = p[2]
+     -sum(dnbinom(x, mu = mu, size = k, log = TRUE))
+ }
```

Method-of-moments estimates, for starting point:

```
> m = mean(x)
> v = var(x)
> mu.mom = m
> k.mom = m/(v/m - 1)
```

Using `optim()` to find best fit:

```
> O1 = optim(fn = NLLfun1, par = c(mu = mu.mom, k = k.mom))
> O1

$par
       mu         k
1.2602356 0.2884793

$value
[1] 71.79646

$counts
function gradient
      45       NA
```

```
$convergence
[1] 0

$message
NULL
```

Calculating likelihood surface:

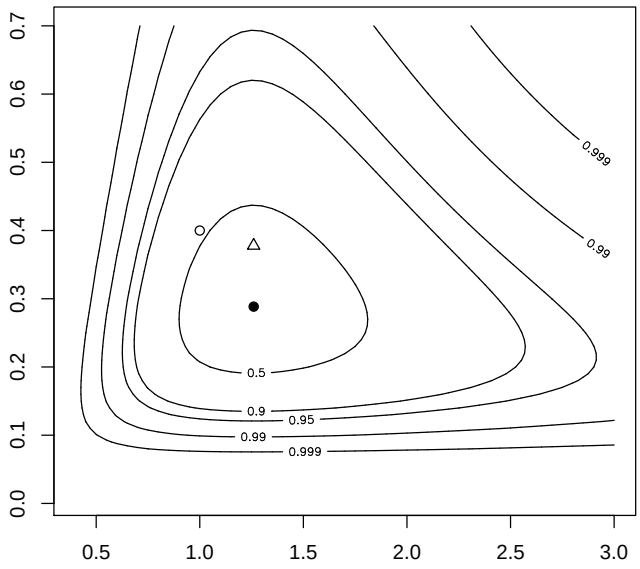
```
> muvec = seq(0.4, 3, by = 0.05)
> kvec = seq(0.01, 0.7, by = 0.01)
> resmat = matrix(nrow = length(muvec), ncol = length(kvec))
> for (i in 1:length(muvec)) {
+     for (j in 1:length(kvec)) {
+         resmat[i, j] = NLLfun1(c(muvec[i], kvec[j]))
+     }
+ }
```

The new part: (1) construct confidence levels (bivariate, so use `df=2` in `qchisq`):

```
> alevels = c(0.5, 0.9, 0.95, 0.99, 0.999)
> minval = O1$value
> nll.levels = qchisq(alevels, df = 2)/2 + minval
```

Draw the contour plot, then add MLEs, true values, and method-of-moments values

```
> contour(muvec, kvec, resmat, levels = nll.levels, labels = alevels)
> points(O1$par["mu"], O1$par["k"], pch = 16)
> points(mu.true, k.true, pch = 1)
> points(mu.mom, k.mom, pch = 2)
```



Exercise 2 :
Set up reef fish data, yet again:

```
> a = 0.696
> b = 9.79
> recrprob = function(x, a = 0.696, b = 9.79) a/(1 + (a/b) * x)
> scoefs = c(mu = 25.32, k = 0.932, zprob = 0.123)
> rzinbinom = function(n, mu, size, zprob) {
+     ifelse(runif(n) < zprob, 0, rnbinom(n, mu = mu, size = size))
+ }
> settlers = rzinbinom(603, mu = scoefs["mu"], size = scoefs["k"],
+     zprob = scoefs["zprob"])
> recr = rbinom(603, prob = recrprob(settlers), size = settlers)
```

Likelihood functions for Shepherd, BH, and constant model, using log-scaled parameters:

```
> NLLfun3L = function(loga, logb, logd) {
+     a = exp(loga)
+     b = exp(logb)
+     d = exp(logd)
+     recrprob = a/(1 + (a/b) * settlers^d)
+     -sum(dbinom(recr, prob = recrprob, size = settlers, log = TRUE),
+         na.rm = TRUE)
```

```
+ }
> NLLfun4L = function(loga, logb) {
+     a = exp(loga)
+     b = exp(logb)
+     recrprob = a/(1 + (a/b) * settlers)
+     -sum(dbinom(recr, prob = recrprob, size = settlers, log = TRUE),
+         na.rm = TRUE)
+ }
> NLLfun5L = function(loga) {
+     a = exp(loga)
+     recrprob = a
+     r = -sum(dbinom(recr, prob = recrprob, size = settlers, log = TRUE),
+         na.rm = TRUE)
+     return(r)
+ }
```

Start by doing linear regression of recruits vs. settlers, with a zero intercept:

```
> lm1 = lm(recr ~ settlers - 1)
> lm.loga = list(log(coef(lm1)))
> rename = function(L, names) {
+     for (i in seq(along = L)) {
+         names(L[[i]]) = NULL
+     }
+     names(L) = names
+     L
+ }
> lm.loga = rename(lm.loga, "loga")
```

First fit density-independent model, using BFGS

```
> library(stats4)
> m5L = mle(minuslogl = NLLfun5L, start = list(loga = -1), method = "BFGS",
+     control = list(ndeps = 0.01))
```

Had to use Nelder-Mead instead of BFGS:

```
> library(stats4)
> m4L = mle(minuslogl = NLLfun4L, start = list(loga = log(0.5),
+     logb = log(10)), method = "Nelder-Mead")
```

set new starting condition to coeffs of last fit:

```
> s3 = c(coef(m4L), list(logd = 0))
```

```
> m3L = mle(minuslogl = NLLfun3L, start = s3, method = "Nelder-Mead")
```

Table of coefficients and log-likelihoods:

```
> lin = c(exp(coef(m5L)), NA, NA, -logLik(m5L))
> BH = c(exp(coef(m4L)), NA, -logLik(m4L))
> shep = c(exp(coef(m3L)), -logLik(m3L))
> ptab = rbind(lin, BH, shep)
> colnames(ptab) = c("a", "b", "d", "NLL")
> ptab
```

```
             a        b        d      NLL
lin  0.1956194       NA       NA 1444.717
BH   0.6469000 9.661556       NA 1020.843
shep 0.6958790 7.488366 0.938981 1020.427
```

Confidence intervals:

```
> exp(confint(m3L))
```

```
Profiling...
         2.5 %     97.5 %
loga 0.5803975  0.8629401
logb 4.5138376 13.2413092
logd 0.8183324  1.0738117
```

```
> exp(confint(m4L))
```

```
Profiling...
         2.5 %     97.5 %
loga 0.5833116  0.7180536
logb 8.9412988 10.4843910
```

```
> exp(confint(m5L))
```

```
Profiling...
    2.5 %    97.5 %
0.1888597 0.2025078
```

Exercise 3 : ZINB, negative binomial, Poisson:

```
> NLLpois = function(lambda) {
+     -sum(dpois(settlers, lambda = lambda, log = TRUE))
+ }
> NLLnb = function(mu, k) {
+     -sum(dnbinom(settlers, mu = mu, size = k, log = TRUE))
+ }
```

Set up ZINB function again:

```
> dzinbinom = function(x, mu, size, zprob, log = FALSE) {
+     v = ifelse(x == 0, zprob + (1 - zprob) * dnbinom(0, mu = mu,
+         size = size), (1 - zprob) * dnbinom(x, mu = mu, size = size))
+     if (log)
+         return(log(v))
+     else v
+ }

> NLLzinb = function(mu, k, zprob) {
+     -sum(dzinbinom(settlers, mu = mu, size = k, zprob = zprob,
+         log = TRUE))
+ }
```

Fit all three functions, from simplest to most complex:

```
> m = mean(settlers)
> mpois = mle(minuslogl = NLLpois, start = list(lambda = m))
> mnbinom = mle(minuslogl = NLLnb, start = list(mu = m, k = 0.5))
> mzinbinom = mle(minuslogl = NLLzinb, start = list(mu = m, k = 0.5,
+     zprob = 0.5))
```

Table of coefficients and log-likelihoods:

```
> pois = c(coef(mpois), NA, NA, -logLik(mpois))
> nbin = c(coef(mnbinom), NA, -logLik(mnbinom))
> zinb = c(coef(mzinbinom), -logLik(mzinbinom))
> ptab = rbind(pois, nbin, zinb)
> colnames(ptab) = c("lambda/mu", "k", "zprob", "NLL")
> ptab
```

```
     lambda/mu         k     zprob      NLL
pois  21.52405        NA        NA 8623.188
nbin  21.52405 0.6238752        NA 2432.672
zinb  24.45607 1.0250050 0.1199043 2411.265
```

The zero-inflated negative binomial wins by a very large margin (how small would we have to make the data set before we couldn't tell the difference any more?) — 6000 log-likelihood units between neg bin and Poisson, and an additional 21 log-likelihood units between neg bin and z-i neg bin ...

```
> confint(mpois)
```

```
Profiling...
   2.5 %   97.5 %
21.15587 21.89647
```

```
> confint(mnbinom)
```

```
Profiling...
        2.5 %     97.5 %
mu 19.4557009 23.8917798
k   0.5553659  0.6993166
```

```
> confint(mzinbinom)
```

```
Profiling...
            2.5 %     97.5 %
mu    22.37720768 26.7320557
k      0.87239529  1.1904082
zprob  0.08789511  0.1532577
```

As expected, confidence limits for μ get wider and wider as we add more parameters to the model. Still, even `zprob` is pretty well bounded from these data.