

# MATH 4Q03/6Q03

## NUMERICAL METHODS FOR DIFFERENTIAL EQUATIONS

### (Winter 2011)

**Time & Place:**

- Lectures — 9:30–10:20 on Mondays & Thursdays, 10:30–11:20 on Tuesday in DSB AB/102
- Computer Labs — 10:30–11:20 on Fridays in BSB/241 & BSB/249

**Instructor:** Dr. Ramesh Yapalparvi

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Office Hours: Monday & Tuesday 1:30–2:30

**Teaching Assistant:** To be announced.**Course Webpage:** <http://www.math.mcmaster.ca/~ramesh/MATH4Q03>

**Outline of the Course:** This course will provide an overview of classical solution methods for ordinary and partial differential equations. The focus will be on finite-difference techniques, but time permitting, certain more advanced approaches such as finite element and spectral methods will also be presented. At the beginning we will also review interpolation and approximation techniques as well as methods of numerical differentiation and integration. As an illustration of the presented algorithms we will use simple MATLAB codes to solve certain computational problems commonly arising in Physics and Engineering.

**Topics:** Numbers in *italic* refer to sections of the textbook by Gerald & Wheatley, whereas numbers in **bold** refer to sections of the textbook by Grasselli & Pelinovsky

## 1. Introduction &amp; Review

- Types of errors in numerical computation (**1.8**) (*0.4-0.5*)
- Root finding methods for nonlinear equations (**8.1-8.2**) (*1.1-1.6*)
- Numerical linear algebra (**2.3-2.7**) (*2.1-2.5*)

## 2. Interpolation &amp; Approximation

- Polynomial interpolation (**5.2-5.4**) (*3.1, 3.2*)
- Spline & Hermite interpolation (**12.1, 12.2**) (*3.3, 3.4*)
- Errors in polynomial interpolation (**5.5**)
- Least squares approximations (**5.6, 5.7**) (*3.6*)

## 3. Numerical differentiation and integration

- Forward, backward and central difference approximation of derivatives (**6.1, 6.2**) (*5.1*)
- Newton-Cotes integration formulas (**6.5, 6.7**) (*5.2, 5.3, 5.6*)
- Gaussian integration formulas (**6.8**) (*5.6*)

## 4. Ordinary differential equations: initial-value problems

- Single-step Euler and Runge-Kutta methods (**9.1, 9.2, 9.3**) (*6.1, 6.2, 6.3*)
- Multi-step explicit and implicit Adams methods (**9.4**) (*6.4, 6.6*)

## 5. Ordinary differential equations: boundary-value problems

- Finite-Difference method (**10.1**) (*6.7*)
- Shooting methods (**10.2**) (*6.7*)

## 6. Finite-difference methods for partial differential equations

- Parabolic partial differential equation (**10.3**) (*8.2*)

- Hyperbolic partial differential equation **(10.4)** (8.3)
  - Elliptic partial differential equation **(10.5)** (8.1)
7. Advanced numerical methods
- Spectral methods **(11.1–11.3)**
  - Finite–element methods **(12.3)** (9.2,9.3)

**Primary Reference:**

1. C. F. Gerald & P. O. Wheatley, “Applied numerical analysis”, Pearson, (2004)

**Secondary Reference:**

1. Numerical Mathematics, by Matheus Grasselli and Dmitry Pelinovsky (Jones and Bartlett Publishers 2008), ISBN 9780763737672 0-13-065243-1

**Additional Reference:**

1. Applied Numerical Methods for Engineers and Scientists, by S. S. Rao (Prentice Hall Publishers 2001)

**Software:** All of the computational examples will be presented using MATLAB. This software is available on the computers in the computer lab. Lab hours (see above) are reserved for unsupervised work with computer–based assignments. Unless they are reserved for large–class tutorials, students should be able to work in the computer labs in BSB also outside the allocated time–slots. Students are as well encouraged to purchase “The Student Edition of MATLAB” to be able to work with MATLAB at home. While a limited introduction to MATLAB will be provided, it is recommended that students grasp basic MATLAB programming skills working either in the computer labs or with the personal student edition.

**Prerequisites:** Calculus, Ordinary and Partial Differential Equations (MATH 3C03 or 3FF3), Numerical Algebra (MATH 2T03), Basic programming skills in MATLAB

**Assignments:** Five home assignments will be handed out in class every second week, starting from January 14. Solutions of the assignments should be submitted by e-mail to [math4q03@math.mcmaster.ca](mailto:math4q03@math.mcmaster.ca) using the template provided. The assignments are due by midnight on Friday the following week. *Late* submissions will not be accepted under any circumstances. Only five best assignments are counted towards the final mark. The assignments and solutions will be posted on the course webpage.

#	Post Date	Due Date
HW 1	Friday, January 14	Friday, January 21
HW 2	Friday, January 28	Friday, February 4
HW 3	Friday, February 11	Friday, February 18
HW 4	Friday, March 4	Friday, March 11
HW 5	Friday, March 18	Friday, March 25

**Midterm Exam:** There will be one midterm test, on the following (tentative) date:

Thursday, February 17

The topics covered on the tests will be announced in class. Students must report any schedule conflict to the course co-ordinator within the first two weeks of classes. You must bring your ID to each test. Only standard McMaster calculators Casio Fx 991 are allowed on the test. A student who has an excuse, approved by the Associate Deans office, for failing to write a test, should consult her/his instructor.

**Final Exam:** The course will be completed by a three–hour final examination. The date and location of the final exam will be announced by the Registrar’s office in mid–term.

**Marking scheme:**

- Final exam (3 hrs) — 50%
- Midterm exam (60 min) — 25%
- Five homework assignments — 25%

**Senate Policy Statement:** The course is regulated by the following documents: *Statement on Academic Ethics* and *Senate Resolutions on Academic Dishonesty*. Any student who infringes one of these resolutions will be treated according to the published policy. In particular, academic dishonesty includes: (1) plagiarism, e.g. the submission of work that is not one's own, (2) improper collaboration in group work on home assignments, (3) copying or using unauthorized aids tests and examinations. It is your responsibility to understand what constitutes academic dishonesty, referring to *Academic Integrity Policy*.

**Important Notice :** The instructor and university reserve the right to modify elements of the course during the term. The university may change the dates and deadlines for any or all courses in extreme circumstances. If either type of modification becomes necessary, reasonable notice and communication with the students will be given with explanation and the opportunity to comment on changes. It is the responsibility of the student to check their McMaster email and course websites weekly during the term and to note any changes.

**Requests for Relief for Missed Academic Term Work:**

- **For absences from classes lasting up to 5 days:**

Using the McMaster student absence form (MSAF) on-line, self-reporting tool, undergraduate students may report absences lasting up to 5 days and may also request relief for missed academic work. The submission of medical or other types of supporting documentation is normally not required. Students may use this tool to submit a maximum of two requests for relief of missed academic work per term. Students must immediately follow up with their course instructors regarding the nature of the relief. Failure to do so may negate the opportunity for relief. It is the prerogative of the instructor of the course to determine the appropriate relief for missed term work in his/her course.

- **For absences from classes lasting more than 5 days:**

Students who are absent more than five days cannot use the on-line, self-reporting tool to request relief. They MUST report to their Faculty Office to discuss their situation and may be required to provide appropriate supporting documentation. If warranted, students will be approved to use a discretionary version of the MSAF on-line, self-reporting tool.