MATH 3GR3 Midterm Test #1

Midterm Test
Duration of test: 50 minutes
McMaster University
October 17, 2023

Instructor: Matt Valeriote

Last Name:______________________________ First Name:_______________

Student Number:______________________________________

Please answer all five questions. To receive full credit, provide justifications for your solutions. For all questions, write your answers in the answer booklet that has been provided. Please be sure to include your name and student number on all sheets of paper that you hand in.

NOTE: In your solutions you may make use of any theorems or results discussed in the lectures. You may not use other theorems or results, unless you fully justify them. This includes results from the homework assignments.

No aids are allowed.

Each question is worth 5 points; the maximal number of marks is 25.

Score

Question	1	2	3	4	5	Total
Score						

[5] 1. (a) Give the definition of a group.

(b) Let $\mathbb{R}' = \{r \in \mathbb{R} : r \geq 0\}$. Let $\diamond$ be the binary operation on $\mathbb{R}'$ defined by $r \diamond s = |r - s|$. Is $\mathbb{R}'$ with the operation $\diamond$ a group? Justify your answer.

2. Let $k > 1$ be an integer and let G be a group with identity element e. Define H_k to be the following subset of G:

$$H_k = \{g \in G : g^k = e\}.$$

[5]

(a) Show that if G is abelian, then H_k is a subgroup of G.

(b) Find an example of a non-abelian group G such that the subset H_2 of G is **not** a subgroup of G.

To receive full credit, explain why the subset H_2 is not a subgroup of the group G that you provide.

[5] 3. Let $\sigma = (1,\ 7,\ 2,\ 5,\ 4)(2,\ 5,\ 3,\ 4)(1,\ 5,\ 6,\ 4)$, a member of the group S_7.

(a) Express σ as a product of **disjoint** cycles.

(b) What is the order of σ?

(c) Is σ an even permutation? Justify your answer to receive credit.

[5] 4. (a) List the elements of $U(14)$, the group of units in $\mathbb{Z}_{14}$.

(b) Is $U(14)$ a cyclic group?

(c) List **all** of the subgroups of $U(14)$.

5. Let G be a group that has at least two elements and that has no proper non-trivial subgroups.

[5]

(a) Show that G must be a cyclic group.

(b) Show that G must be a finite group and that $|G|$ is a prime number.