

Math 3TP3 Truth and Provability Term 1 Autumn 2014–2015
Assignment 6 due December 3, 2014

- 1) Write a Turing machine program to compute the function $f(x, y) = x + y$. Your program should assume that it begins with a tape that contains only the values of x and y (in binary, of course) separated by one B , and that it is scanning the first 1 of the input x . It should end with the scanner looking at the first 1 of the output $x + y$. First write the algorithm, then list the quadruples of the program.
- 2) Show that the decidability of the generalised word problem for groups $w_1 = w_2$, where w_1 and w_2 are words in a finitely presented group, is equivalent to the specific problem $w_1 = 1$.
- 3) Write $\mathbb{Z}[i]$ for the complex integers; that is, the complex numbers $a + bi$ where both a and b are integers. Show that if there were an algorithm to decide $p(X) = 0$, where $p(X)$ is a polynomial over $\mathbb{Z}[i]$ and the solution is sought in $\mathbb{Z}[i]$, then there would also be an algorithm to solve Hilbert's Tenth Problem. Deduce that solvability for polynomials in $\mathbb{Z}[i]$ is undecidable.