Math 3TP3 Truth and Provability Term 1 Autumn 2014–2015
Assignment 4
recommended for October 29, but formally due November 4, 2014

1) Recall that Gödel's β-function is defined by

$$\beta(c, d, i) = \text{rem}(c, d(i+1)+1) = \text{ the remainder when } c \text{ is divided by } d(i+1)+1.$$

  a) Suppose a sequence has $c = 5435$ and $d = 6$. Find all elements of the sequence.
  b) Find the c and d that code the sequence $k_0 = 2$, $k_1 = 3$. Give your calculations, and verify that your values of c and d work as claimed.

2) Suppose we used instead the β-function defined by

$$\beta(c, i) = \text{exf}(c, i) = \text{the highest power of } \pi_i \text{ that divides } c,$$

  where π_i is the ith prime in the list of primes in order starting with $\pi_0 = 2$.
  a) Find the sequence coded by 45000.
  b) For the sequence $k_0 = 2$, $k_1 = 3$, find c.

3) This problem is about expressing functions in $\mathcal{L}_A$, the language of arithmetic. Recall that $\mathcal{L}_A$ includes function symbols for the unary successor function S and the binary functions $+$ and $\cdot$. Thus the formula $\text{Succ}(x, y)$ that expresses the relation $y = Sx$ is immediately an $\mathcal{L}_A$-formula, as is the formula $\text{Prod}(x, y, z)$ that expresses the relation $z = x \cdot y$.
  a) Find $\mathcal{L}_A$ formulas that express the following functions (notice that these functions are defined by primitive recursion, but the formulas do not need the full complexity of the p.r. definition).
      i) $\text{Prec}(x, y)$ if and only if $y = Px$, where P is defined by $P0 = 0$, $P(Sx) = x$.
      ii) $\text{Minus}(x, y, z)$ if and only if $z = x \ominus y$, where $x \ominus 0 = x$, $x \ominus Sy = P(x \ominus y)$.
  b) The function $|x - y| = x \ominus y + y \ominus x$ is defined by a composition involving $+$ and $\ominus$. Find a an $\mathcal{L}_A$-formula that expresses $\text{Abs}(x, y, z)$ if and only if $z = |x - y|$.
  c) Find a formula that expresses $\text{Fact}(x, y)$ if and only if $y = x!$ (you will have to follow through the discussion in class of how to express a function defined by primitive recursion).