Practice Problems for Midterm II
Math 2R03 Autumn 2007–08

1) Define the following terms: *linear transformation, kernel, image, one-to-one, onto, isomorphism, invariant subspace, eigenspace, characteristic polynomial.*

2) Let $T : V \to V$ be a linear operator, and let u, v be eigenvectors in V associated to different eigenvectors. Prove that $\{u, v\}$ is linearly independent.

3) Let $T : \mathbf{P}_2 \to \mathbf{P}_2$ be the linear transformation defined by $T(p(x)) = p(x) + p(-x)$. Find $\ker(T)$ and $\mathrm{im}(T)$.

4) Calculate the change-of-basis matrix $P_{B \leftarrow E}$ where E is the standard basis and B is the basis $\{1, x-2, x^2+3\}$ for $\mathbf{P}_2$. Use $P_{B \leftarrow E}$ to express $p(x) = a + bx + cx^2$ in the basis B.

5) Find the matrix M_{EB} for the linear transformation in problem 3), where B is the basis in 4).

6) Consider $A = \begin{pmatrix} 11 & -4 & -5 \\ 21 & -8 & -11 \\ 3 & -1 & 0 \end{pmatrix}$. Find the eigenvalues of A. For each eigenvalue, find its generalised eigenspace. Show directly that each generalised eigenspace is an invariant subspace. Find a basis of $\mathbf{R}^3$ with respect to which A has block diagonal form, and each block is upper triangular. Find the matrix P such that $P^{-1}AP$ is the block diagonal matrix of A.