Practice Problems for Midterm 1

Math 2R03 Autumn 2007–08

1) Give the precise definition of the following phrase: *a set of vectors in a vector space V is a basis for V.*

2) State whether the following is TRUE or FALSE: *If u is in the span of $\{v_1, \ldots, v_n\}$ then $\{u, v_1, \ldots, v_n\}$ is dependent.*

3) With careful reference to the axioms for a vector space, show that $(01)v = -v$ for any vector v in a vector space.

4) Let X, Y be subsets of a vector space V such that $\mathrm{span}(X) = \mathrm{span}(Y) = V$. Is it necessarily the case that $x \cap Y \neq \emptyset$? Justify your answer.

5) Find a basis for $\mathbf{R}^4$ which contains the following set of vectors: $\{(0, 0, 3, 2), (0, 1, 7, 4)\}$. Justify your answer.

6) Consider $U = \{p(x) \in \mathbf{P}_4 : p(x) = p(-x) \text{for all } x \in \mathbf{R}\}$. Show that U is a subspace. Find a basis for U, and hence its dimension.

7) Let V be the set of 2×2 matrices with equal column sums. Show that V is a subspace of $\mathbf{M}_{22}$. Find a basis for V and calculate its dimension.

8) Find a basis for $\mathbf{M}_{22}$ consisting of matrices with the property that $A^2 = A$.