Project: Q4.

$$s_1 F_1(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} s_1 e^{3t} \cos(2t) \\ -s_1 e^{3t} \sin(2t) \\ 0 \end{pmatrix}$$

steps:

$$\begin{pmatrix} \cos(2t) \\ -\sin(2t) \end{pmatrix}$$

$$e^{3t} \begin{pmatrix} \cos(2t) \\ -\sin(2t) \end{pmatrix}$$

$$s_1 e^{3t} \begin{pmatrix} \cos(2t) \\ \sin(2t) \end{pmatrix}$$

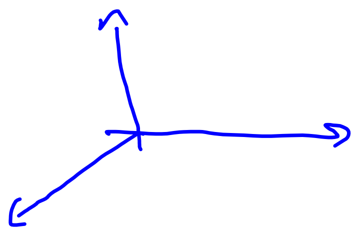
$s_2 F_2(t)$ — same.

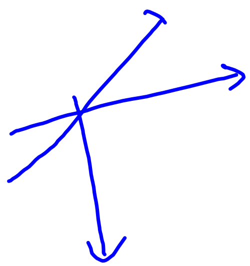
$\boxed{s_1 F_1(t) + s_2 F_2(t)}$ — same.

$$\boxed{s_3 F_3(t)} = \begin{pmatrix} 0 \\ 0 \\ \cancel{s_3} e^{t} \end{pmatrix}$$

$$A = P^{-1} M A$$

$$\underbrace{e^{At}}_{} = \underbrace{P^{-1} e^{Mt} P}_{?}$$





Principal Axis Thm.

$T: V \to V$

V has an orthogonal basis of eigenvectors of T iff T is symmetric.

Let U be a subspace of inner product space V.

$U^\perp = \{ v \in V : \langle v, u \rangle = 0 \text{ for all } u \in U \}$.

$U =$ x-axis in $\mathbb{R}^3$; then $U^\perp =$ y-z plane.

Theorem Let $T: V \to V$ be a symmetric linear operator, and suppose that U is a T-invariant subspace. Then $T: U \to U$ is a symmetric operator and $U^\perp$ is T-invariant.

Pf. Since U is T-invariant then T is a function on U; and the defn of symmetric holds for all $u_1, u_2 \in U$.

Consider $v \in U^\perp$. We want to show that $T(v) \in U^\perp$.

Consider $\langle T(v), u \rangle$ for any $u \in U$.

$$\langle T(v), u \rangle = \langle v, T(u) \rangle \quad \text{as } T \text{ symmetric}$$

$$= \langle v, u' \rangle \quad \text{for some } u' \in U \text{ as } U\ T\text{-inv.}$$

$$= 0, \text{ as } v \in U^\perp.$$

Thus $T(v) \in U^\perp$ as required. $\blacksquare$

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