

10.2Orthogonal Lemma:

Let $\{e_1, \dots, e_m\}$ be an orthogonal set of vectors in an inner product space. Let v be any vector which is not in $\text{span}\{e_1, \dots, e_m\}$

Define

$$e_{m+1} = v - \frac{\langle v, e_1 \rangle}{\|e_1\|^2} e_1 - \dots - \frac{\langle v, e_m \rangle}{\|e_m\|^2} e_m$$

Then $\{e_1, \dots, e_{m+1}\}$ is an orthogonal set of vectors.