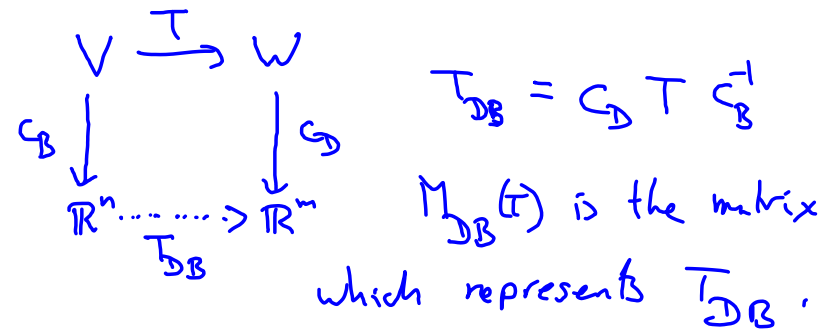


Lecture 19: Change of Basis



$$T_{DB}(X) = M_{DB} X.$$

$$M_{DB} = (C_D T(e_1) \quad C_D T(e_2) \quad \dots \quad C_D T(e_n))$$

$$T = C_D^{-1} T_{DB} C_B$$

For any vector $\vec{v} \in V$,

$$T(\vec{v}) = C_D^{-1} (T_{DB}(C_B(\vec{v})))$$

$$C_D(T(\vec{v})) = T_{DB}(C_B(\vec{v}))$$

$$C_D(T(\vec{v})) = M_{DB} C_B(\vec{v}) \quad (*)$$

Consider $T = Id: V \rightarrow V$.

~~$$T(\vec{v}) = Id(\vec{v}) = \vec{v}.$$~~

$$(*) \text{ says: } C_D(\vec{v}) = M_{DB} C_B(\vec{v})$$

Consider $a + bx + cx^2$

$$\mathcal{C}_D(a + bx + cx^2) = \mathcal{P} \leftarrow \mathcal{D} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$= \begin{pmatrix} a + b + c \\ b + 2c \\ c \end{pmatrix}$$

$$\mathcal{C}_D^{-1} \mathcal{C}_D(a + bx + cx^2) = (a + b + c)1 + (b + 2c)(x - 1) + c(x - 1)^2$$

Given $T: V \rightarrow V$, B, D two bases for V . We have:

$$c_D(v) = P_{D \leftarrow B} c_B(v)$$

$$c_B(T(v)) = M_B(T) c_B(v)$$

$$c_D(T(v)) = M_D(T) c_D(v)$$

How do M_B and M_D compare?

$$\begin{aligned} \underbrace{P_{D \leftarrow B} c_B(T(v))}_{c_D(T(v))} &= c_D(T(v)) \\ &= M_D(T) c_D(v) \\ &= M_D(T) P_{D \leftarrow B} c_B(v) \end{aligned}$$

$P_{D \leftarrow B} M_B(T) c_B(v)$ Since this holds for all vectors v ,

$$\text{then } P_{D \leftarrow B} M_B = M_D P_{D \leftarrow B}$$

$$\text{ie. } M_B = P_{D \leftarrow B}^{-1} M_D P_{D \leftarrow B}$$

M_B and M_D are similar matrices.