Project I will be due on Tuesday Oct 16 in class.

refs: § 6.6

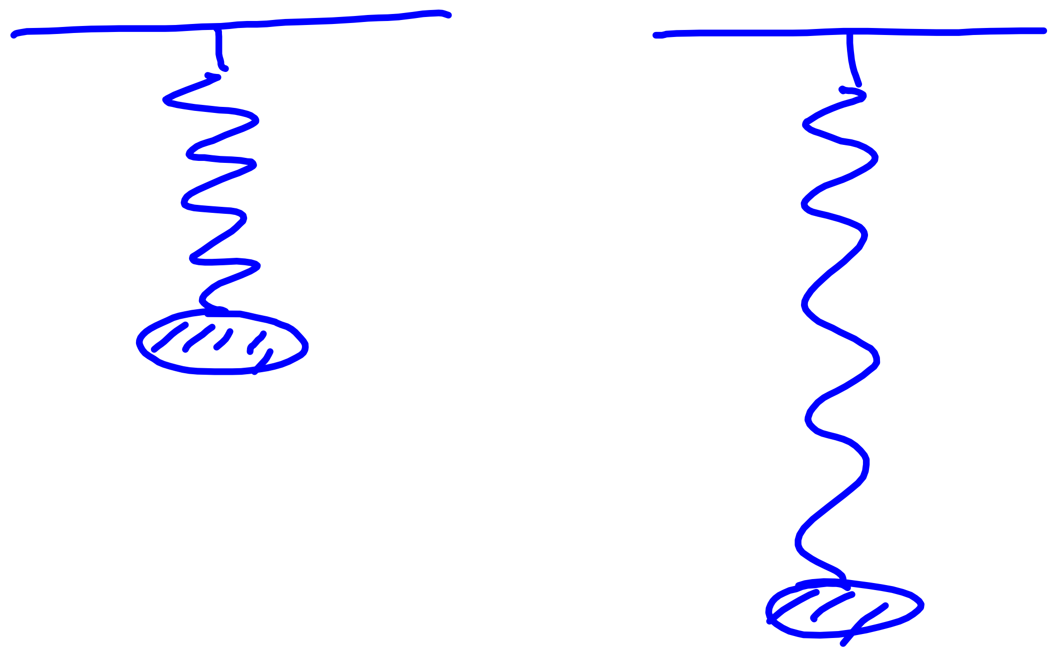
Taylor series

to be done in groups of 1-4 people

Assessment of the group contribution:

rating:
3 contributed fully
2 less complete contribution
1 no ideas to contribute
0 wrote their name on it.

$$\text{Mark} = \text{proj. mark} \times \frac{\text{avg assessment}}{3}$$



oscillates : sines/cosines

cases: ideal : oscillates forever

real → 1) oscillations decrease

2) very viscous – no oscillation

Ex. $T: M_{22} \to M_{22}$

$T(X) = XA - AX$, where $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(T is linear).

$$T\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} b-c & a-d \\ d-a & c-b \end{pmatrix}$$

$$\text{im}(T) = \left\{ \begin{pmatrix} x & y \\ -y & -x \end{pmatrix} : x, y \in \mathbb{R} \right\}.$$

Basis for $\text{im}(T)$: $\begin{pmatrix} x & y \\ -y & -x \end{pmatrix} = \begin{pmatrix} x & 0 \\ 0 & -x \end{pmatrix} + \begin{pmatrix} 0 & y \\ -y & 0 \end{pmatrix}$

$$= x\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + y\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Thus $\left\{ \underset{w_1}{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}, \underset{w_2}{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} \right\}$ is a spanning set for $\text{im}(T)$; this set is lin. ind., hence a basis. $\dim(\text{im}(T)) = 2$

$$\ker(T) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : \begin{pmatrix} a-d & b-c \\ c-b & d-a \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \right\}$$

Thus $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \ker(T)$ if and only if

$a = d$ and $b = c$.

$$\ker(T) = \left\{ \begin{pmatrix} a & b \\ b & a \end{pmatrix} : a, b \in \mathbb{R} \right\}$$

$$\begin{pmatrix} a & b \\ b & a \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Thus $\left\{ \overset{v_3}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}, \overset{v_4}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} \right\}$ is a spanning set for $\ker(T)$; independent so also a basis.

$\dim(\ker(T)) = 2$.

$$w_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = T\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right) = T(v_1)$$

$$w_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = T\left(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right) = T(v_2)$$

$\{v_1, v_2, v_3, v_4\}$ is a basis for M_{22}.

$\dim(\operatorname{im}(T)) + \dim(\ker(T)) = \dim(V)$.