

Math 1XX3 Winter 2013 Maple Lab 2

Instructions This lab is about using Maple to plot slope fields and solution curves of differential equations, and to plot curves given parametrically or in polar coordinates. Read through Sections 3.2, 3.4, 7.3, 8.1, 8.2 of the CalcLab manual. Then do the following problems. You should hand in a Maple worksheet (with your name on it) that includes your commands and the solutions to the problems (*clearly indicate* where to find the solutions). Clean up the worksheet so that any mistakes you made in the process are eliminated from view.

1) There is considerable evidence to support the theory that for some species there is a minimum population m such that the species will become extinct if the population falls below m . This condition can be incorporated into the logistic equation by introducing the factor $P - m$. The modified logistic model is then given by the differential equation

$$\frac{dP}{dt} = k(M - P)(P - m) .$$

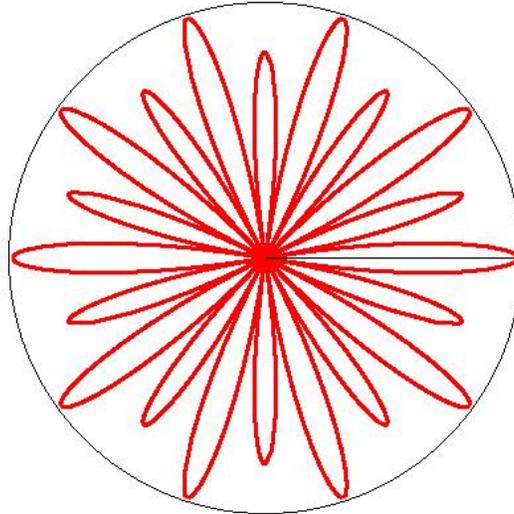
For this problem, take the constants to be $M = 400, m = 200, k = .08$

- i) With the constants given, plot the slope field for the differential equation, along with the solution curves for the initial conditions $P(0) = 170, P(0) = 195, P(0) = 205, P(0) = 230,$ and $P(0) = 550$. Explain what these curves tell you about the behaviour of the population over time for different initial conditions.
- ii) Find the explicit solution to the differential equation (using Maple!) which satisfies $P(0) = 115$.
- iii) It can be seen from the original differential equation, and is illustrated in the above slope field, that if the initial population size is less than m , then the population will be decreasing and the species will eventually become extinct. (This can also be shown analytically from the explicit solution.) For the explicit solution you calculated in (ii), find the time t at which the species goes extinct.

2) Consider a Lissajous figure given by $x = \cos(3t), y = \sin(t)$, for $0 \leq t \leq 2\pi$, which consists of three distinct “lobes”. Find the length of the middle lobe (i.e., the portion of the curve that is between $y = -0.5$ and $y = 0.5$ when you plot the curve). You will have to experiment with different ranges for t to figure out which range(s) produce the required part of the curve. To help with this, you should first work out (by hand), what values of t satisfy $x(t) = 0$.

3) The picture below was produced using the maple command

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polarplot(1 + a * cos(b * theta), theta = 0..2 * Pi, tickmarks = [0, 0]);
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where a and b are integers greater than 1.

- i) Fix $a = 2$ and try different values of b . Describe the results.
- ii) Fix $b = 2$ and try different values of a . Describe the results.
- iii) Let both a and b vary. Make a general statement about the shape of the curve depending on the values of a and b .
- iv) What values of a and b were used to produce the picture above?