

Math 1C03: Introduction to Mathematical Reasoning

Practice Problems for Midterm

- 1) Know your definitions! These include *necessary*, *sufficient*, *equivalent*, *union*, *intersection*, *cartesian product*, *set complement*, *injective*, *surjective*, *bijective*, *countable*. Knowing a definition means both being able to state it precisely and being able to give examples.
- 2) Express the following statements in symbolic form:
 - (a) For a real number x to have a cube root it is necessary that it be positive.
 - (b) Every natural number which has remainder 1 after division by 4 can be written as a sum of squares of four natural numbers.
 - (c) Every integer greater than 8 can be written in the form $3x + 5y$ for some non-negative integers x and y .
 - (d) Let p be a prime. If every positive integer r less than p satisfies $r^p = qp + 1$ for some integer q then the same is true for any positive integer.
- 3) Prove 2(c) above by induction.
- 4) Use the division algorithm to prove 2(d).
- 5) Suppose A and B are countably infinite sets. Prove that $A \times B$ is also countably infinite.
- 6) Find the solution set of the inequality $|(x - 1)(x + 2)| > 3$.
- 7) Prove by induction that a function from a finite set to itself is injective if and only if it is surjective.