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The poincaré conjecture:

Topology: Every closed compact 3-dimensional manifold, where every continuous loop is shrinkable to a point is homeomorphic to the 3-dimensional sphere.

Geometry: study ~~shapes~~, angles, not even with coordinates necessarily, size, measurement

Topology: study of shapes without concern for size.

Poincaré conjecture is a problem in topology, but solved using geometric methods.

S^1 circle  1-dimensional

S^2 sphere  2-dimensional

near a point looks like a 2-dim plane

Has no boundary – this is what "closed" means above.

Möbius strip: 2-dim surface with boundary, but boundary is a single circle.

Non-oriented.

Cut in half – introduces a new boundary, but is still in one piece.

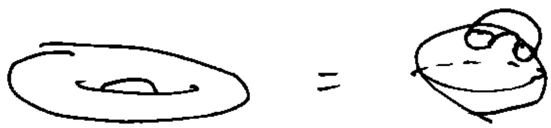
Note: connection with DNA. Needs to divide in two but helical shapes can make this difficult – it becomes linked.

	vertices	edges	faces
cube	8	12	6
icosahedron	12	30	20

$V-E+F=2$

For any 2-dim surface "shaped like" a sphere, $V-E+F=2$. This is called the Euler characteristic. It is a topological invariant – does not change under deformation.

Other closed surfaces



$V-E+F=0$

$V-E+F=2-2g$.

\# holes called the genus

Determining the genus completely determines the surface. i.e. every compact closed orientable 2-manifold is determined just by its genus.

Is there a similar list for all 3-dimensional surfaces?

A continuous loop is a path on the surface.

On surface with genus 0, any loops can be shrunk to a point. On surface with genus > 0, there are loops that cannot be shrunk to a point.

<u>Theorem</u> the S^2 sphere is the only 2-dim manifold on which every loop can be shrunk to a point.

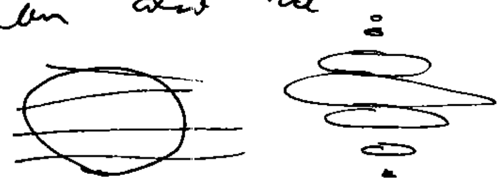
<u>Poincaré</u> Same true for 3 dimensions?

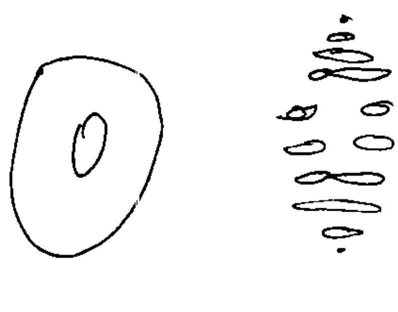
S^2: take 2-dim. disk and collapse boundary to a point.

(Can do the same for a doughnut).

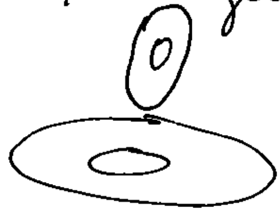
S^3: take 3-dim ball and collapse boundary to a point.

Can also see S^2 as a sequence of slices:





this is the fundamental idea of string theory.

Can I view S^3 as the gluing together of two solid doughnuts:  glue boundary.

G. Perelman: picture of all 3-manifolds.
(2003)

He refused the millenium prize money.