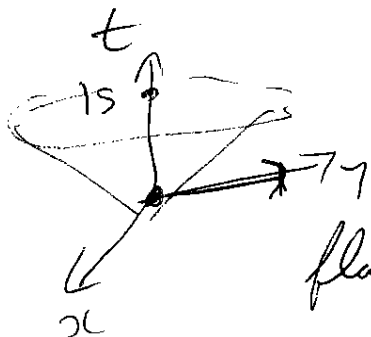


T. Howard Yang-Mills Existence and Mass Gap

more detailed review by Jaffe-Witten on Clay website.
 Newton 1687 → Einstein 1905.

\mathbb{R}^4 - space and time



flashlight beam set at from origin along
 y axis and after this $t=1$ be at

$$y = 3 \times 10^8 \text{ m} = \text{speed of light} \times 1.$$

Possible directions for red light beam

$$\text{forms a cone: } x^2 + y^2 + z^2 - c^2 t^2 = 0$$

$$\text{Let } x_1 = x, x_2 = y, x_3 = z, x_4 = ict.$$

then symmetric in x_1, x_2, x_3, x_4 :

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = \text{euclidean distance.}$$

Newtonian mechanics → wrong at the atomic scale:
 quantum mechanics invented in 1920's by
 Bohr, Dirac, Heisenberg, Schrödinger

Intuition no longer works at the quantum mechanical level.

Quantum Field theory QFT

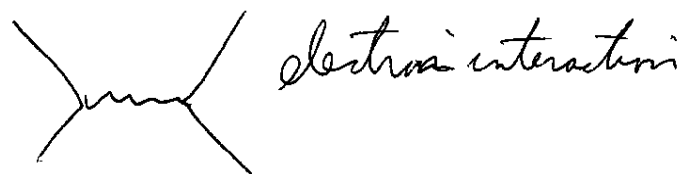
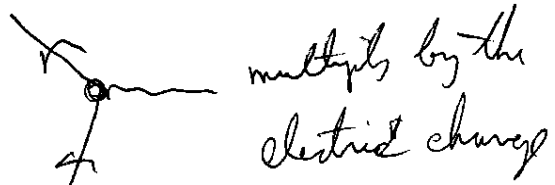
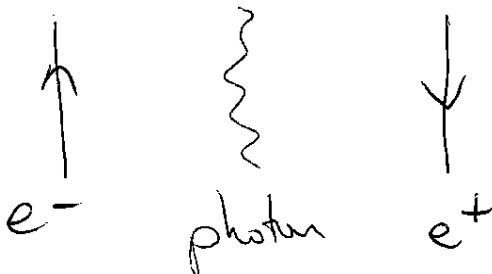
Particles: electrons, neutrons, protons
muons, neutrinos, π

Forces: gravity, electricity, magnetism
weak and strong nuclear forces

1865 - Maxwell understood that electricity and magnetism are really the same, and that light waves are also really the same thing.

Quantum Electrodynamics QED

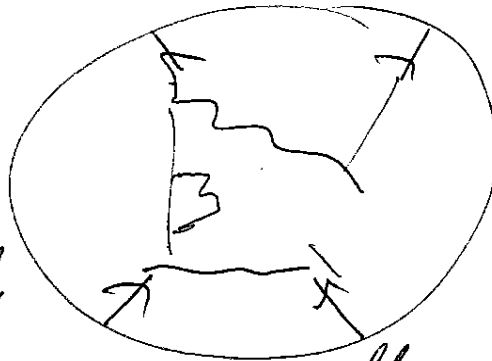
Feynman diagrams: to understand QFT.



Add up everything: i.e. draw all possible pictures of the interaction of two electrons.

Each interaction

you corresponds to an integral, then take



a quadruple integral to average the whole picture over all of space.

You get: $\int_{-\infty}^{\infty} = \infty$ - it doesn't make sense.

So renormalize: $\int_{-\infty}^{\infty} + \int_{-\infty}^{\infty} = \text{finite!}$

QED gives the best predictions of experimental data.

QED is an example of a gauge theory: G is a circle, which forms a group, a continuous group.

$$R_{\theta} \circ R_{\theta'} = R_{\theta+\theta'} = R_{\theta'} \circ R_{\theta}$$

circle is $\{z \in \mathbb{C} : |z|=1\}$ properties: complex multⁿ.

Yang Mills theory has a gauge group G which

$$= SU_n(2) = \left\{ u = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{C} \right\}$$

$\det(u) = 1, u^\dagger = u^{-1}$

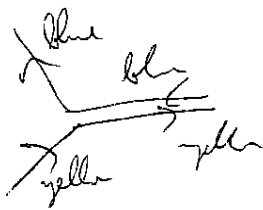
Group operation is matrix multiplication, which is not commutative.

Yang-Mills theory attempts to explain the nuclear forces which hold the nucleus of an atom together.

Idea: proton is made up of a bunch of quarks which come in colors — mixed together they give white.

Quantum Chromodynamics: 3-colours, described by $SU(3)$.

Interaction between quarks given by gluons, which themselves have colour combinations.



Now put all these together, as we did with QED.

— very complicated calculations.

Pure Yang-Mills: gluons without quarks

Problem: cannot see the separate quarks in a proton.

Heuristic: cannot see colours. The "strong force" holds the colours / quarks together. This strong force is called the ~~Mass~~ Mass Gap.