Math 1C Homework # 6

§5.6 1/ defg.

(d) $R = \{ (m,n) \in \mathbb{N} \times \mathbb{N} : m^2 = n \}$

$= \{ (m, m^2) : m \in \mathbb{N} \}.$

This is a function. domain is $\mathbb{N}$
range is the perfect squares in $\mathbb{N}$

(e) $R = \{ (m,n) \in \mathbb{Z} \times \mathbb{Z} : m^2 = n \}$

$= \{ (m, m^2) : m \in \mathbb{Z} \}$

This is a function. domain is $\mathbb{Z}$
range is the positive integers which are perfect squares.

(f) $R = \{ (m,n) \in \mathbb{N} \times \mathbb{N} : n^2 = m \}$

$= \{ (n^2, n) \in \mathbb{N} \times \mathbb{N} \}$

This is a function. (as $n = \sqrt{m}$, but there is not the option of a negative square root).
domain is the perfect squares in $\mathbb{N}$
range is $\mathbb{N}$.

(g) $R = \{ (m,n) \in \mathbb{Z} \times \mathbb{Z} : n^2 = m \}$.

This is not a function, as for example,

$(4,2) \in R$ and $(4,-2) \in R$.

2 bc

(b) $\{ (x,y) \in \mathbb{R} \times \mathbb{R} : y = \tan(x) \}$

domain $= \{ x \in \mathbb{R} : x \neq n\frac{\pi}{2}, n \in \mathbb{Z} \}$.

range $= (-\infty, \infty)$.

(c) $\{ (x,y) \in \mathbb{R} \times \mathbb{R} : y = \frac{1}{x-1} \}$

domain $= (-\infty, 1) \cup (1, \infty)$.

range $= (-\infty, 0) \cup (0, \infty)$

§5-8

2/ x is not the lub of A if

$\neg \left(\forall a \in A \, (a \leq x) \ \& \ \forall \varepsilon > 0 \, \exists a \in A \, (a > x - \varepsilon) \right)$

equiv: $\exists a \in A \, (a > x) \vee \exists \varepsilon > 0 \, \forall a \in A \, (a \leq x - \varepsilon)$

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3/ $\text{lub}(a,b) = b$.

$\text{lub}\,[a,b] = b$

$\max\,[a,b] = b$

$\max\,(a,b)$ does not exist

4/ $A = \{ |x-y| \,;\, x,y \in (a,b) \}$

Observe that A ~~gives~~ is the set of possible distances between elements of the interval (a,b). Thus A is bounded above by $|b-a|$. In fact, $A = [0, |b-a|)$. Thus $\text{lub}\,A = |b-a|$.

§59

2/ $\{a_n\} = \left\{ \left(\frac{n}{n+1}\right)^2 \right\}$.

To prove $a_n \to 1$, need to show that,

$\forall \varepsilon > 0\ \exists N\ \forall n > N \left(1 - \left(\frac{n}{n+1}\right)^2 < \varepsilon \right)$.

(using the observation that $\left(\frac{n}{n+1}\right)^2 < 1$).

Fix ε. We need to find N so that for all $n > N$, $1 - \left(\frac{n}{n+1}\right)^2 < \varepsilon$.

$$1 - \left(\frac{n}{n+1}\right)^2 = \frac{(n+1)^2 - n^2}{(n+1)^2}$$

$$= \frac{2n+1}{(n+1)^2}$$

$$< \frac{2n}{(n+1)^2}$$

$$< \frac{n}{(n+1)^2} = \frac{1}{n+1} \cdot \frac{n}{n+1}$$

If $n \geq 2$ then $\frac{n}{n+1} > \frac{1}{2}$, so

$1 - \left(\frac{n}{n+1}\right)^2 < \frac{1}{2} \cdot \frac{1}{(n+1)}$. Thus it suffices

if $\frac{1}{2} \cdot \frac{1}{(n+1)} < \varepsilon$ i.e. $\frac{1}{n+1} < 2\varepsilon$

$n+1 > \frac{1}{2\varepsilon}$.

Thus, ~~~~ take $N = \text{integer part}\left(\frac{1}{2\varepsilon}\right)$.

Then for all $n > N$, by the above calculation,

$$1 - \left(\frac{n}{n+1}\right)^2 < \varepsilon.$$

5/ the sequence $\{a_n\}$ tends to infinity if /5

$$\forall M > 0 \ \exists N > 0 \ \forall n > N \ (a_n > M).$$

(for any possible bound M, there is an N such that all later values of the sequence are greater than the value M).

$\{n\}$ tends to ∞: given $M \in \mathbb{R}$ take

$$N = ip(M) + 1.$$

$\{2^n\}$ tends to ∞: given $M \in \mathbb{R}$, take

$$N = ip(\log_2(M)).$$