Math 1C Homework #1

sec 2.1 p19

(1) 1b) $(3<4) \wedge (3<6)$ is equivalent to $(3<4)$ ✓

2a) $(\pi^2 > 2) \Longrightarrow (\pi > 1.4)$ (T), as both antecedent and consequent are true

(4) b) $(\pi^2 < 0) \Longrightarrow (\pi = 3)$ (T) as antecedent is false,

c) $(\pi^2 > 0) \Longrightarrow (1+2=4)$ (F) the antecedent is true, but the consequent is false

d) $(\pi < \pi^2) \Longrightarrow (\pi = 5)$ (F) the antecedent is true, but the consequent is false

sec 2.2 p26

2) A is necessary for B equivalent to $B \Rightarrow A$

(6) B " n is divisible by 6"

a) $B \Rightarrow$ n is divisible by 3, so this condition is necessary

b) $B \not\Rightarrow$ n is divisible by 9 not necessary

c) $B \not\Rightarrow$ n div. by 12 not necessary

d) $n=24 \not\Rightarrow B$ not necessary

e) $B \Rightarrow n^2$ div. by 3 necessary

f) $B \Rightarrow$ n is even and divisible by 3 necessary

[3] A is sufficient for B is $A \Rightarrow B$.

(6)

a) n div by 3 $\not\Rightarrow$ n div by 6 — not sufficient

b) n div by 9 $\not\Rightarrow$ n div by 6 — not sufficient

c) n div by 12 $\Rightarrow$ n div by 6 — sufficient

d) $n = 24 \Rightarrow$ n div by 6 — sufficient

e) n^2 div by 3 $\not\Rightarrow$ B — not sufficient

f) n even and div by 3 $\Rightarrow$ B — sufficient

(2) [4] necessary and sufficient: (f)

[9a)]

(2)

ϕ	ψ	$\phi \Rightarrow \psi$	$\neg(\phi \Rightarrow \psi)$	$\phi \wedge \neg\psi$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

Since the last two columns are the same, the formulae are equivalent.

[11)]

(2)

ϕ	ψ	$\neg\phi$	$\neg\psi$	$\phi \Rightarrow \psi$	$\neg\psi \Rightarrow \neg\phi$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

[13] ϕ unless ψ means ϕ holds unless ψ holds. p3

(3) Thus ϕ holds whenever ψ does not, and if ψ holds, then ϕ does not. Thus

ϕ unless ψ is equivalent to $(\phi \vee \neg\psi) \wedge (\psi \rightarrow \neg\phi)$ ($\neg\psi \rightarrow \phi$)

or $(\neg\psi \wedge \phi) \vee (\psi \wedge \neg\phi)$

or $\boxed{\phi \Leftrightarrow \neg\psi}$

never on $\neg\phi \Leftrightarrow \psi$

[14]

(8)

	Antecedent	Consequent
a)	apples are red	oranges are green
b)	f is differentiable	f is continuous
c)	f is integrable	f is bounded
d)	s is convergent	s is bounded
e)	2^{n-1} is prime	n is prime
f)	the team wins	Karl is playing
g)	Karl plays	team wins
h)	Karl plays	team wins

[15]

(4) d) converse: s is bounded implies s is convergent

contrapositive: if s is not bounded then s is not convergent

f) converse: if Karl played then the team won

contrapositive: if Karl did not play then the team did not win.

sec 2.3 p32

1 b) 1,000,000 is not the largest natural number

① $\exists x\,(x \in \mathbb{N} \;\&\; x > 1{,}000{,}000)$.

2 c) the eqn $x^3 = 28$ does not have a natural number solution

① $\forall x\,(x \in \mathbb{N} \rightarrow x^3 \neq 28)$

3 a) everybody loves somebody

② $\forall x\,\exists y\,(x \text{ loves } y)$

d) nobody is at home

$\neg \exists x\,(x \text{ is at home})$

4 a) $\forall a \in \mathbb{R}\,\exists x \in \mathbb{R}\,(x^2 + a = 0)$

⑤ b) $\forall a \in \mathbb{R}\,(a < 0 \rightarrow \exists x \in \mathbb{R}\,(x^2 + a = 0))$

c) $\forall x \in \mathbb{R}\,\exists p \in \mathbb{N}\,\exists q \in \mathbb{N}\,(x = p/q)$

d) $\exists x \in \mathbb{R}\,\forall p \in \mathbb{N}\,\forall q \in \mathbb{N}\,(x \neq p/q)$ $\quad I(x)$

e) ~~$\forall p_1\,\forall q_1\,\exists p_2\,\exists q_2\,(\frac{p_1}{q_1} < \frac{p_2}{q_2})$~~

$\forall x \in \mathbb{R}\,(I(x) \rightarrow \exists y \in \mathbb{R}\,(I(y) \;\&\; y > x))$.

sec 2.4 p 39

(1) b) $\exists x\,(x \in F \;\&\; x \notin C)$

F = set of my friends
C = people who have cars

(3)

f) $\forall x \exists y\,(x \text{ loves } y)$

j) $\neg \forall x\,(P(x) \rightarrow B(x))$

$P(x)$ x is precious
$B(x)$ x is beautiful

(2) b) $\exists x \in \mathbb{N}\,(2x+3 = 5x+1)$

(7) $2x+3 = 5x+1 \iff 3x \cdot 2 = 3x$ F
$\iff x = 2/3$

c) $\exists x' \in \mathbb{Q}\,(x^2 = 2)$ F $\sqrt{2}$ is irrational

i) $\exists x^{\in \mathbb{R}} \forall y\,(x+y=0)$ F cannot have $x=-y$ for all y.

j) $\forall x^{\in \mathbb{R}} \exists! y\,(y=x^2)$ T the square of a number is unique

k) $\forall x^{\in \mathbb{N}} \exists! y\,(y=x^2)$ T

o) $\forall x^{\in \mathbb{R}}(x<0 \rightarrow \exists y\,(y^2=x))$ F

p) $\forall x^{\in \mathbb{R}}(x<0 \rightarrow \exists y\,(y^2=x))$ T because there is no positive real number which satisfies the antecedent.

3) b) ~~$\neg \exists x \in \mathbb{N}$~~

— $\neg \exists x\ (x \in F \ \&\ x \notin C)$

$\forall x\ (x \in F \to x \in C)$

(3) all my friends have cars.

f) $\neg\ \forall x\ \exists y\ (x \text{ loves } y)$

— $\exists x\ \forall y\ (x \text{ does not love } y)$

Somebody hates everybody.

j) $\forall x\ (P(x) \to B(x))$

— all precious stones are beautiful

5) (a) $\neg\ (\forall x \in \mathbb{N})\ \exists y \in \mathbb{N}\ (x+y = 1)$

— $\exists x \in \mathbb{N}\ \forall y \in \mathbb{N}\ (x+y \neq 1)$

(4)

(b) $\neg\ \forall x > 0\ \exists y < 0\ (x+y = 0)$

— $\exists x > 0\ \forall y \geq 0\ (x+y \neq 0)$

(c) $\neg\ \exists x\ (\forall \varepsilon > 0)\ (-\varepsilon < x < \varepsilon)$

— $\forall x\ \exists \varepsilon > 0\ (x \geq \varepsilon \vee x \leq -\varepsilon)$

(d) $\neg\ \forall x \in \mathbb{N}\ \forall y \in \mathbb{N}\ \exists z \in \mathbb{N}\ (x+y = z^2)$

— $\exists x \in \mathbb{N}\ \exists y \in \mathbb{N}\ \forall z \in \mathbb{N}\ (x+y \neq z^2)$

6) $\neg\ \forall \varepsilon > 0\ \exists \delta > 0\ \forall x\ (|x-a| < \delta \to |f(x)-f(a)| < \varepsilon)$

(1) $\exists \varepsilon > 0\ \forall \delta > 0\ \exists x\ (|x-a| < \delta\ \&\ |f(x)-f(a)| \geq \varepsilon)$.

Extra Credit

2-2 (20) vowel on side A ⇒ even number on side B

Rule would be violated if 1) a card had a vowel on the visible side and odd number on the other side

(2) or 2) a card had an odd number on the visible side and a vowel on the other side.

Thus need to turn over the E and the 7.

(21) Cannot see ID's: check the beer drinker

(2) Can see the ID's: check the drink of the under-age drinker.

(22) Rule is if underage ⇒ can't drink alcohol.

(2) Same logical structure.
But second one is easier because we understand, without expressing the rule, what the issue is. Whereas with the first, we don't know what the issue is.

sec 2.3 (7) $F(x,t)$ means person x is fooled at time t

(2) $\exists x \forall t F(x,t) \;\&\; \exists t \forall x F(x,t) \;\&\; \neg \forall x \forall t F(x,t)$.

sec 2.4 (7) $\neg \big(\exists x \forall t F(x,t) \;\&\; \exists t \forall x F(x,t) \;\&\; \neg \forall x \forall t F(x,t) \big)$

(2) equiv to: $\exists x \forall t F(x,t) \;\&\; \exists t \forall x F(x,t) \longrightarrow \forall x \forall t F(x,t)$.