Math 1A03 Midterm 2 2011

1. (a) $\int \cos(3x)\,dx = \int \frac{1}{3}\cos(u)\,du = \frac{1}{3}\sin(u) + C$

subst. $u = 3x$
$du = 3\,dx$

$\boxed{= \frac{1}{3}\sin(3x) + C}$

(b) $\int x e^{x^2}\,dx = \int \frac{1}{2} e^u\,du = \frac{1}{2}e^u + C =$

subst $u = x^2$
$du = 2x\,dx$

$\boxed{= \frac{1}{2}e^{x^2} + C}$

(c) $\int \frac{\ln x}{x}\,dx = \int u\,du = \frac{u^2}{2} + C = \boxed{\frac{(\ln x)^2}{2} + C}$

subst $u = \ln x$
$du = \frac{1}{x}\,dx$

(d) $\int \frac{1}{x^2+9}\,dx = \boxed{\frac{1}{3}\arctan\left(\frac{x}{3}\right) + C}$

(from the formula sheet, or we use $u = \frac{x}{3}$)

2. $f^{(4)}(x)$ is positive and increasing on $[0,1]$, so

$$|f^{(4)}(x)| \le f^{(4)}(1) \quad \text{for all } x \text{ in } [0,1]$$

and we can take

$$K = f^{(4)}(1) \approx 3479$$

The error in Simpson's rule satisfies

$$|E_s| \le \frac{K(b-a)^5}{180n^4}$$

We want $|E_s| \le 10^{-1}$, so we take n such that

$$\frac{K(b-a)^5}{180n^4} \le 10^{-1}$$

$$n^4 \ge \frac{K(b-a)^5}{18}$$

$$n \ge \sqrt[4]{\frac{K(b-a)^5}{18}} \approx 3.73$$

Therefore it is enough to take $\boxed{n=4}$ (note that it is an even number which we need to apply Simpson's rule)

Then $$\int_0^1 e^{e^x}dx \approx S_4 = \frac{\Delta x}{3}\left(f(0)+4f(1/4)+2f(2/4)+4f(3/4)+f(1)\right)$$

$(\Delta x = 1/4)$ ≈ 6.33 correct to within 10^{-1}

3. $\displaystyle\int \frac{-x^2+3x+8}{x^3-4x^2+4x}\,dx$

$$x^3-4x^2+4x = x(x^2-4x+4) = x(x-2)^2$$

$$\frac{-x^2+3x+8}{x^3-4x^2+4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \quad / \cdot x(x-2)^2$$

$$-x^2+3x+8 = A(x-2)^2 + Bx(x-2) + Cx$$

Option 1

$x=0 \Rightarrow 8 = A\cdot 4 \Rightarrow \underline{A=2}$

$x=2 \Rightarrow 10 = C\cdot 2 \Rightarrow \underline{C=5}$

$x=1 \Rightarrow 10 = A - B + C$

$10 = 2 - B + 5$

$\underline{B=-3}$

Option 2

$-x^2+3x+8 = (A+B)x^2 + (-4A-2B+C)x + 4A$

equating coefficients

(i) $A+B=-1$

(ii) $-4A-2B+C=3$

(iii) $4A=8$

(iii) $\Rightarrow \underline{A=2}$

(i) $\Rightarrow 2+B=-1 \Rightarrow \underline{B=-3}$

(ii) $\Rightarrow -8+6+C=3 \Rightarrow \underline{C=5}$

So $\displaystyle\int \frac{-x^2+3x+8}{x^3-4x^2+4x}\,dx = \int\left(\frac{2}{x} - \frac{3}{x-2} + \frac{5}{(x-2)^2}\right)dx$

$$\boxed{= 2\ln|x| - 3\ln|x-2| - \frac{5}{x-2} + C}$$

(the integrals $\int \frac{3}{x-2}\,dx$ and $\int \frac{5}{(x-2)^2}\,dx$ are solved by substitution $u=x-2$)

4/7

4. (a) $\int x \sin x \, dx = x(-\cos x) - \int -\cos x \, dx = \boxed{-x\cos x + \sin x + C}$

by parts $u = x$ $\quad dv = \sin x \, dx$

$du = dx$ $\quad v = -\cos x$

(b) $\int_0^{\pi} \sin x \cos x \sin(\cos x) \, dx = \int_1^{-1} -u \sin u \, du$

subst $u = \cos x$

$du = -\sin x$

when $x = 0$ $\quad u = 1$

$x = \pi$ $\quad u = -1$

$= -\left[-u\cos u + \sin u\right]_{+1}^{-1}$ (by (a))

$= -\left(\left(\cos(-1) + \sin(-1)\right) - \left(-\cos(1) + \sin(1)\right)\right)$

$= -\cos(-1) - \cos(1) - \sin(-1) + \sin(1)$

$= 2\sin(1) - 2\cos(1)$

$\boxed{\approx 0.60}$

5/7

5. $$\int \frac{1}{x^2\sqrt{x^2-36}}\,dx = \int \frac{6\tan\theta\sec\theta}{(6\sec\theta)^2\sqrt{(6\sec\theta)^2-36}}\,d\theta$$

subst $x = 6\sec\theta$ $\quad 0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$

$dx = 6\tan\theta\sec\theta\,d\theta$

$$= \int \frac{\tan\theta\sec\theta}{6\sec^2\theta\sqrt{36(\sec^2\theta-1)}}\,d\theta$$

$\sec^2\theta - 1 = \tan^2\theta$

$$= \frac{1}{36}\int \frac{\tan\theta\sec\theta}{\sec^2\theta\sqrt{\tan^2\theta}}\,d\theta$$

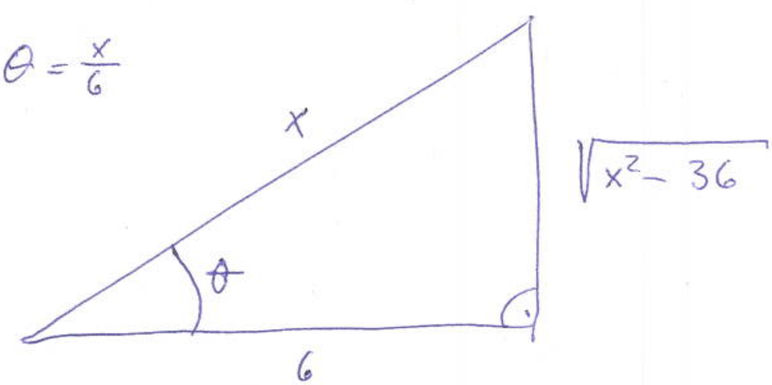
$$= \frac{1}{36}\int \frac{\tan\theta\sec\theta}{\sec^2\theta\tan\theta}\,d\theta =$$

$$= \frac{1}{36}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{36}\int \cos\theta\,d\theta$$

$$= \frac{1}{36}\sin\theta + C$$

$$\boxed{= \frac{1}{36}\,\frac{\sqrt{x^2-36}}{x} + C}$$

$\sec\theta = \frac{x}{6}$



6/7

6.(a) $\displaystyle\int_1^4 \sqrt{x}\ln(3x)\,dx =$

by parts

$u = \ln(3x) \qquad dv = \sqrt{x}\,dx$

$du = \frac{1}{3x}\cdot 3\,dx \qquad = x^{1/2}\,dx$

$= \frac{1}{x}\,dx \qquad v = \frac{2}{3}x^{3/2}$

$$= \left[\frac{2}{3}x^{3/2}\ln(3x)\right]_1^4 - \int_1^4 \frac{2}{3}x^{3/2}\cdot\frac{1}{x}\,dx$$

$$= \quad '' \quad - \frac{2}{3}\int_1^4 x^{1/2}\,dx = \left[\frac{2}{3}x^{3/2}\ln(3x)\right]_1^4 - \frac{2}{3}\left[\frac{2}{3}x^{3/2}\right]_1^4$$

≈ 9.409

(b) $$\int \frac{x+3}{x^2+6x+10}\,dx = \frac{1}{2}\int \frac{2x+6}{x^2+6x+10}\,dx =$$

subst $u = x^2+6x+10$
$du = (2x+6)\,dx$

$$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

$$\boxed{= \frac{1}{2}\ln|x^2+6x+10| + C}$$

alternatively,

$$\int \frac{x+3}{x^2+6x+10}\,dx = \int \frac{x+3}{(x+3)^2+1}\,dx = \int \frac{u}{u^2+1}\,du = \int \frac{1}{2w}\,dw =$$

sub $u = x+3$ $\qquad$ subst $w = u^2+1$
$du = dx$ $\qquad$ $dw = 2u\,du$

$$= \frac{1}{2}\ln|w| + C = \frac{1}{2}\ln|u^2+1| + c = \boxed{\frac{1}{2}\ln|(x+3)^2+1| + C}$$

6. (c)

$$\int e^{x^3+2\ln x}\,dx = \int e^{x^3}\cdot e^{2\ln x}\,dx =$$

$$= \int e^{x^3}\cdot\left(e^{\ln x}\right)^2 dx = \int e^{x^3}\cdot x^2\,dx =$$

subst. $u = x^3$
$du = 3x^2\,dx$

$$= \frac{1}{3}\int e^u\,du$$

$$= \frac{1}{3}e^u + C = \boxed{\frac{1}{3}e^{x^3} + C}$$

(d) $$\int \tan^4 t\,\sec^4 t\,dt = \int \tan^4 t\,\sec^2 t\cdot\sec^2 t\,dt =$$

$$= \int \tan^4 t\,(\tan^2 t + 1)\,\sec^2 t\,dt$$

subst. $u = \tan t$
$du = \sec^2 t\,dt$

$$= \int u^4(u^2+1)\,du =$$

$$= \int (u^6 + u^4)\,du = \frac{u^7}{7} + \frac{u^5}{5} + C = \boxed{\frac{\tan^7 t}{7} + \frac{\tan^5 t}{5} + C}$$