ArtSci 1D06 Calculus 2017–2018
Practice questions for Winter Midterm 3

This is a list of practice questions in order to prepare for Midterm 3. It represents the approximate difficulty and approximate length of the actual exam.

1)

a) State precisely what it means to say that the sequence $\{a_n\}$ diverges.

b) State precisely what it means to say that the sequence $\{a_n\}$ is increasing.

c) State precisely what it means to say that the series $\sum_{n=0}^{\infty} a_n$ diverges.

d) State precisely what it means to say that the series $\sum_{n=0}^{\infty} a_n$ is bounded.

e) State the alternating series test for convergence of the series $\sum_{n=0}^{\infty} (-1)^n b_n$, where $b_n > 0$ for all n.

2)

a) Write the formula for the Taylor series around a for a function $f(x)$.

b) Use your answer to a) to find the Taylor series for the function $f(x) = (1 - 3x)^{1/2}$ around 0. (Do not just quote a known Taylor series.)

c) Find the radius of convergence of this series.

3)

a) State the divergence test.

b) Let $\{a_n\}$ be a decreasing sequence such that $\lim\limits_{n\to\infty} a_n = \dfrac{1}{2}$. Write $s_m = \sum\limits_{n=1}^{m} a_n$. Find a lower bound for s_m (this will depend on m). Deduce that $\sum\limits_{n=1}^{\infty} a_n$ diverges (thus verifying the divergence test for this example).

4)

a) State the monotone sequence theorem.

b) Show that the sequence $\{ne^{-n}\}$ converges.

5) The Taylor series around 0 for the function e^x is $\sum_{n=0}^{\infty} \frac{1}{n!} x^n$.

a) Use this to find the Taylor series around 0 for the function e^{-x^2}.

b) Hence find a Taylor series for $\int e^{-x^2}\, dx$.