Name: ____________________
Student number: ____________________

ARTSCI 1D06

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DAY CLASS
DURATION OF EXAMINATION 2.5 Hours
MCMASTER UNIVERSITY MIDYEAR EXAMINATION

Thursday 17 December 2015

THIS EXAMINATION PAPER INCLUDES SEVEN QUESTIONS ON NINE PAGES. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Special instructions: Answer all the questions in the space provided.
If you need more paper, ask the invigilator.
Use of Casio-FX-991 calculator only is permitted.
This paper must be returned with your answers.

Problem	Points	Problem	Points
1 [10]		**5** [5]	
2 [15]		**6** [10]	
3 [10]		**7** [5]	
4 [5]			
		Total [60]	

1) [10 points]

a) True or false (justify your answer): If a function is continuous on its whole domain, then it is differentiable on its whole domain.

b) True or false (justify your answer): If a function is integrable on its whole domain then it is differentiable on its whole domain.

c) Find $\dfrac{dy}{dx}$ if $x \cos y + y \cos x = 1$

d) Find $\displaystyle\int x\sqrt{x^2+1}\,dx$.

2) [15 points]

a) Find $\displaystyle\int_0^{\infty} xe^{-2x}\ dx$.

b) Find $\displaystyle\int \frac{\sin(x)\cot(x)}{\sec(x)}\ dx$.

c) Find $\int \frac{6x - 11}{x^2 + x - 12}\, dx$.

3) [10 points]

a) Find an approximation to the integral $\int_0^4 \sqrt{7x+9}\, dx$ using a Riemann sum with left endpoints and $n = 4$.

b) Find the exact value of the integral in a).

4) [5 points]

a) State the Intermediate Value Theorem.

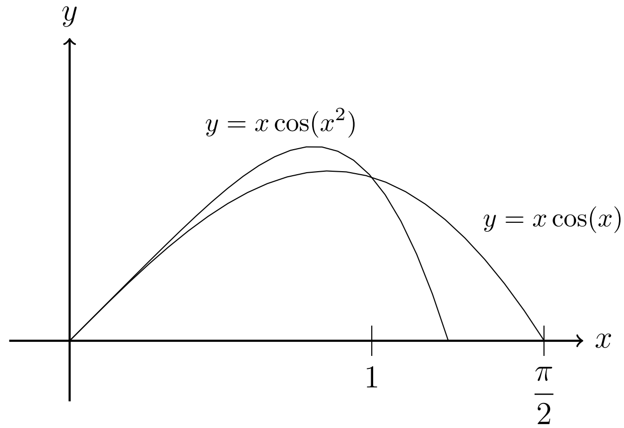
b) Is there a number that is exactly 1 less than its cube? Justify your answer.

5) [5 points] Consider the function

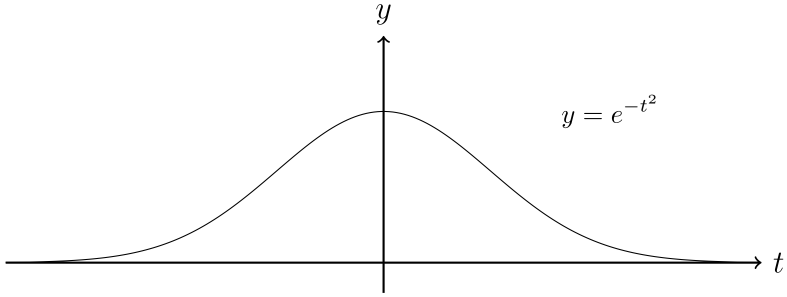
$$h(x) = \begin{cases} \dfrac{2\sin(x)}{x}, & \text{if } x \neq 0; \\ a, & \text{if } x = 0, \end{cases}$$

where a is a constant. Find the value of a so that h is continuous at 0.

6) [10 points] The graphs of $y = x\cos(x^2)$ and $y = x\cos(x)$ are shown below. Find the area between them on the interval $[0, 1]$.



7) [5 points] The function $y = e^{-t^2}$ has graph as shown below.



Consider $f(x) = \int_{-\infty}^{x} e^{-t^2}\, dt$.

a) Sketch a very rough graph of $y = f(x)$ (use the fact that $\lim_{x\to\infty} f(x) = \sqrt{\pi}$). You should do this just by considering the graph of e^{-x^2}; there is no need for any calculation.

b) Does $f(x)$ have any critical points? If so, compute them and mark them on your graph.

c) Does $f(x)$ have any inflection points? If so, compute them and mark them on your graph.

Formula Sheet

Integrals (constants of integration are omitted)

$$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1 \qquad \int \frac{1}{x}\,dx = \ln|x|$$

$$\int e^x\,dx = e^x \qquad \int a^x\,dx = \frac{a^x}{\ln a}$$

$$\int \sin x\,dx = -\cos x \qquad \int \cos x\,dx = \sin x$$

$$\int \tan x\,dx = -\ln|\cos x| \qquad \int \cot x\,dx = \ln|\sin x|$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| \qquad \int \csc x\,dx = -\ln|\csc x + \cot x|$$

$$\int \sec^2 x\,dx = \tan x \qquad \int \csc^2 x\,dx = -\cot x$$

$$\int \sec x \tan x\,dx = \sec x \qquad \int \csc x \cot x\,dx = -\csc x$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) \qquad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)$$

Trigonometry

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

$$\sin(2x) = 2\sin x\cos x \quad \cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$$

Newton's method $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Arc length $L = \displaystyle\int_a^b \sqrt{1 + [f'(x)]^2}\ dx$