ArtSci 1D06 Calculus
Full year 2017–2018
Instructor: D. Haskell

Winter Midterm 4 —Practice version

Instructions There are five questions, and a formula sheet (the same will be given on the actual midterm).

1) a) Sketch the parametric curve given by the equations $x = \sin(t)$, $y = \cos(2t)$ for $0 \le t \le 2\pi$. Indicate the direction in which the curve is traversed.

   b) Find $\dfrac{dx}{dt}, \dfrac{dy}{dt}$ and all points (x, y) where the curve has a horizontal tangent line or a vertical tangent line.

2) Find $\displaystyle \int x^3 \ln(7x)\, dx$.

3) Find $\displaystyle \int \frac{1}{(x^2 + 9)^2}\, dx$.

4) Find $\displaystyle \int_1^\infty \frac{1}{x(x + 2)}\, dx$

5) Find $\displaystyle \int \sin(\sqrt{x})\, dx$.

Formula Sheet

Integrals (constants of integration are omitted)

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}, \ n \neq -1 \qquad\qquad \int \frac{1}{x} \, dx = \ln|x|$$

$$\int e^x \, dx = e^x \qquad\qquad \int a^x \, dx = \frac{a^x}{\ln a}$$

$$\int \sin x \, dx = -\cos x \qquad\qquad \int \cos x \, dx = \sin x$$

$$\int \tan x \, dx = -\ln|\cos x| \qquad\qquad \int \cot x \, dx = \ln|\sin x|$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| \qquad\qquad \int \csc x \, dx = -\ln|\csc x + \cot x|$$

$$\int \sec^2 x \, dx = \tan x \qquad\qquad \int \csc^2 x \, dx = -\cot x$$

$$\int \sec x \tan x \, dx = \sec x \qquad\qquad \int \csc x \cot x \, dx = -\csc x$$

$$\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) \qquad\qquad \int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin\left(\frac{x}{a}\right)$$

Trigonometry

$$\sin^2(x) + \cos^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x) \qquad 1 + \cot^2(x) = \csc^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x) \quad \cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1$$