Name:_____
Student number:_____

ARTSCI 1D06

DEIRDRE HASKELL

DAY CLASS
DURATION OF EXAMINATION 2.5 Hours
MCMASTER UNIVERSITY FINAL EXAMINATION — PRACTICE

Wednesday 25 April 2018

This is a practice version of the final exam. It is intended to give you an idea of the approximate length and difficulty of the actual final. You should not expect the actual exam to include the exact same topics in the same order.

Special instructions: Answer all the questions in the space provided.
If you need more paper, ask the invigilator.
Use of any calculator is permitted.
This paper must be returned with your answers.

1) (10 points) Find the following integrals.

a) $\int \tan(x)\,dx$.

b) $\int \cos(\sqrt{x})\,dx$

c) $\int \frac{x^2}{(4-x^2)^{3/2}}\,dx$

2) (10 points) Find all first and second order partial derivatives of the function $f(x, y) = \dfrac{3x}{x^3 - 4y^2}$.

3) (10 points) Find and classify all the critical points of the function $f(x, y) = x\cos(y)$.

4) (10 points) Find the equation of the tangent plane to the surface $z = f(x, y) = e^x \ln(1 + y)$ at the point $(0, 0)$. Use the tangent plane to find an approximation to the value of $f(0.1, -0.1)$.

5) (10 points) Show that $\lim_{(x,y)\to(0,0)} \dfrac{x^6}{x^6+3y^2}$ does not exist.

6) (10 points) Sketch the curve given by the parametric equations

$$x = t^2, \quad y = \sin(t), \quad -\pi \leq t \leq \pi$$

Find the area enclosed by this curve to the left of $x = \pi^2$.

7) (10 points) Determine whether the following series converge. If the series converges, determine its sum:

(1) $\sum_{n=1}^{\infty} 4^{n-1}$

(2) $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$

8) (10 points) State the definition of the Taylor series for a function $f(x)$ around $x = a$. Then find the Taylor series for $f(x) = (1 - x)^{-2}$ around 0 from the definition (no credit for simply quoting a binomial series!). Find the radius of convergence of your series.

Formula Sheet

Integrals (constants of integration are omitted)

$$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1 \qquad \int \frac{1}{x}\,dx = \ln|x|$$

$$\int e^x\,dx = e^x \qquad \int a^x\,dx = \frac{a^x}{\ln a}$$

$$\int \sin x\,dx = -\cos x \qquad \int \cos x\,dx = \sin x$$

$$\int \tan x\,dx = -\ln|\cos x| \qquad \int \cot x\,dx = \ln|\sin x|$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| \qquad \int \csc x\,dx = -\ln|\csc x + \cot x|$$

$$\int \sec^2 x\,dx = \tan x \qquad \int \csc^2 x\,dx = -\cot x$$

$$\int \sec^3 x\,dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x|$$

$$\int \sec x\tan x\,dx = \sec x \qquad \int \csc x\cot x\,dx = -\csc x$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) \qquad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)$$

Trigonometry

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

$$\sin(2x) = 2\sin x\cos x \quad \cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$$

Arc length $L = \displaystyle\int_a^b \sqrt{1+[f'(x)]^2}\ dx = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$