ArtSci 1D06 Calculus
Full year 2015–2016
Fall Midterm — Practice version

Instructions There are six questions on five pages. Answer all the questions in the space provided. If you need more paper, ask the invigilator.

NAME:

ID NUMBER:

TUTORIAL DAY AND TIME

Problem	Points
1 [10]	
2 [5]	
3 [5]	
4 [5]	
5 [5]	
6 [10]	
Total [40]	

1) [10 points]

a) Find $f'(x)$ for the function $f(x) = \sin(x^2)$.

b) Find $f'(x)$ for the function $f(x) = \dfrac{e^{2x} + e^{3x}}{e^{4x}}$.

c) Find $\dfrac{dy}{dx}$, where $x^2y^2 = \cos(x)$.

d) Find $f'(x)$ for the function $f(x) = x^2 e^x \sinh(x)$.

e) Given $h(x) = f(\sin(x))$, $f(1) = 2$ and $f'(1) = 5$, find $h'(\pi/2)$.

2) [5 points] Consider the function

$$f(x) = \begin{cases} -ax^2 + b, & \text{if } x \leq 1; \\ (x-1)^3 + 2, & \text{if } x > 1. \end{cases}$$

Find a and b so that f is both continuous and differentiable at $x = 1$.

3) [5 points] Let $f(x) = x - \cos(x)$.

a) Use the intermediate value theorem to show that f has at least one zero.

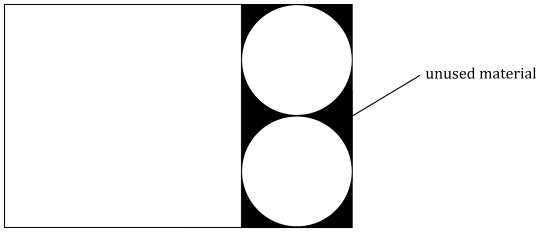
b) Find $f'(x)$ and use it to show that f has exactly one zero.

4) [5 points]

a) State the limit definition of the derivative.

b) Given the function $f(x) = \dfrac{1}{x-3}$ find its derivative f from the definition.

5) [5 points] A cylindrical can has a volume of 100 cubic centimetres. It is formed out of a single rectangular piece of sheet metal by first cutting a rectangle to form the curved surface area, and then cutting circles from the remaining piece to form the top and bottom. Find the dimensions of the can to minimise the wasted material. Assume no material is required to crimp the edges. Unlike the picture, the height of the can does not have to be bounded by the diameters of the circles.



6) [10 points] Let $f(x) = xe^{-x}$.

a) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$, using L'Hôpital's Rule if appropriate.

b) Find and classify all critical numbers of f.

c) Find all inflection points of f.

d) Sketch the graph of $y = f(x)$.