Arts & Science 1D06 Quiz #8 Solutions

January 27, 2016

Full Name:______________________________ Student # :________________

TA:______________________________

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[10 marks] (1.) Determine whether the following series are convergent or divergent.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3-3}}$ *Hint: Use the limit comparison test*

This series is similar to $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3}}$,a convergent p-series, since $p = 3/2 > 1$.

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^3-3}}}{\frac{1}{\sqrt{n^3}}} = \lim_{n\to\infty} \frac{\sqrt{n^3}}{\sqrt{n^3-3}} = \lim_{n\to\infty} \frac{\sqrt{n^3}}{n^{\frac{3}{2}}\sqrt{1-\frac{3}{n^3}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1-\frac{3}{n^3}}} = 1$$

Since 1 is a positive, finite number, $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3}}$ and $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3-3}}$ share the same property. So $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3-3}}$ is convergent as well.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^3+1}$ *Hint: Use the comparison test*

This looks similar to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$, a convergent p-series with $p = 3$, so we would suspect it behaves the same way.

$n^3 + 1 > n^3$, so $\dfrac{1}{n^3+1} < \dfrac{1}{n^3}$, and $0 \le \cos^2 n \le 1$.

So $\dfrac{\cos^2 n}{n^3+1} \le \dfrac{1}{n^3+1} < \dfrac{1}{n^3}$.

Since $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^3+1}$ is less than a known convergent series, we can conclude that this series converges as well by the comparison test.