Arts & Science 1D06 Quiz #7 SOLUTIONS

15 January 2015

Full Name:______________________________ Student # :____________

TA:______________________________

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[3 marks] (1) Find the formula for the general term of this sequence:

$$S = \left\{ \frac{2}{3}, -\frac{5}{9}, \frac{8}{27}, -\frac{11}{81}, \ldots \right\}.$$

Denominator is powers of 3, starting at 3^1. Numerator increases in increments of 3, starting at 2, which is $3(1) - 1$. So the sequence can be written:

$$S = \left\{ (-1)^{n+1} \frac{3n-1}{3^n} \right\}_{n=1}^{\infty}.$$

[3 marks] (2) Show that S is bounded.

This sequence is bounded above by 1 and below by -1. We know that $(3n-1) < 3^n$ for all n, and since both sides of that inequality are positive, we can say:

$$(3n-1) < 3^n \implies \frac{3n-1}{3^n} < 1$$

and

$$(3n-1) < 3^n \implies -\frac{3n-1}{3^n} > -1.$$

In other words,

$$-1 < (-1)^{n+1} \frac{3n-1}{3^n} < 1,$$

and so S is bounded.

[2 marks] (3) Is S convergent? If so, what does it converge to?

Yes, S is convergent to 0. We know this because, if the absolute value of a sequence converges to zero, then the sequence as a whole converges to zero. And

$$\lim_{n\to\infty} \left| (-1)^n \frac{3n-1}{3^n} \right| = \lim_{n\to\infty} \frac{3n-1}{3^n} = 0,$$

since exponentials grow much faster than linear terms.

[2 marks] (4) If a sequence is convergent, is the series formed by summing the terms of that sequence necessarily convergent?

No way! Consider the sequence $\{1, 1, 1, \ldots\}$ No doubt it's convergent, though the sum of all these terms goes off to ∞.