Arts & Science 1D06 Quiz #7 Solutions

January 13, 2016

Full Name:______________________________ Student # :________________

TA:______________________________

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[6 marks] (1.) Consider the sequence

$$\left\{-\frac{4}{3}, 1, -\frac{16}{27}, \frac{25}{81}, -\frac{36}{243}, \ldots\right\}$$

(a) Find a formula for the general term a_n of the sequence.

$$a_n = (-1)^n \frac{(n+1)^2}{3^n}$$

(b) Does the sequence converge? If so, to what value?

Yes, it converges to 0. Use that if $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$ and that we can relate limits of sequences to limits of functions.

$$\lim_{x\to\infty} \frac{(x+1)^2}{3^x} \overset{\text{L.H.}}{=} \lim_{x\to\infty} \frac{2(x+1)}{(3^x)(\ln 3)} \overset{\text{L.H.}}{=} \lim_{x\to\infty} \frac{2}{(3^x)(\ln 3)^2} = 0$$

So we have $\lim_{n\to\infty} \left|(-1)^n \frac{(n+1)^2}{3^n}\right| = \lim_{x\to\infty} \frac{(x+1)^2}{3^x} = 0 = \lim_{n\to\infty} (-1)^n \frac{(n+1)^2}{3^n}$

[4 marks] (2.) Suppose you know that b_n is an increasing sequence and all its terms lie between the numbers 3 and 5.

(a) Explain why the sequence must have a limit.

Since b_n is an increasing sequence, $b_n < b_{n+1}$ for all $n \geq 1$. Because all its terms lie between 3 and 5, b_n is a bounded sequence. By the Monotonic Sequence Theorem, b_n is convergent, ie b_n has a limit, L.

(b) What can you say about the value of the limit?

L must be greater than 3 since b_n is increasing, so $3 < L \leq 5$.