Arts & Science 1D06 Quiz #12

31 March 2016

Full Name: SOLUTIONS Student # :

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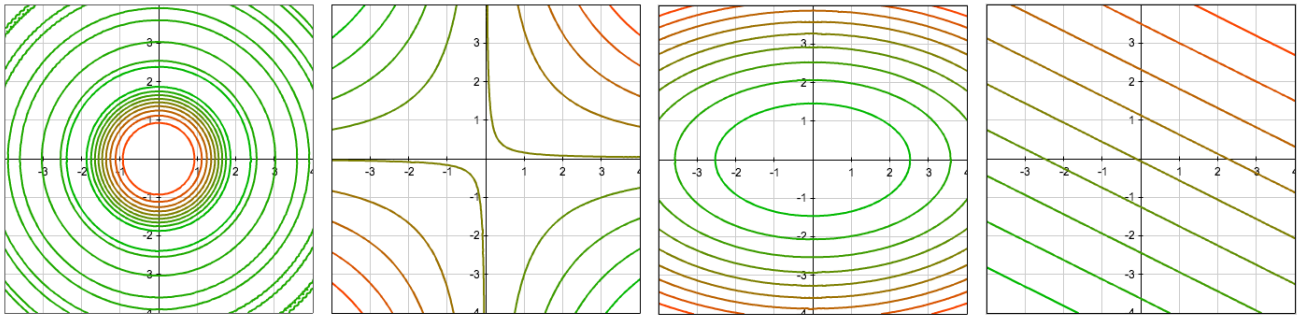
Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

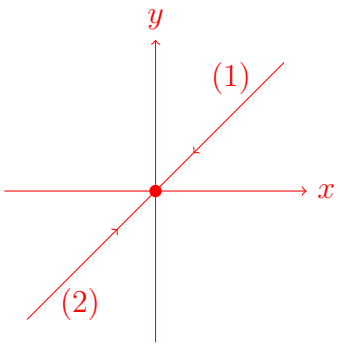
[4 marks] (1) Match the contour map with the formula of the function

$$A.\ f(x,y) = x + 2y \quad B.\ g(x,y) = \frac{\sin(x^2+y^2)}{x^2+y^2} \quad C.\ h(x,y) = xy \quad D.\ k(x,y) = x^2 + 3y^2$$



$A. \leftrightarrow IV.$ $B. \leftrightarrow I.$ $C. \leftrightarrow II.$ $D. \leftrightarrow III.$

[6 marks] (2) Show that $\lim_{(x,y)\to(0,0)} \frac{\sqrt{xy}}{x^2+y^2}$ does not exist.



We'll use the two approaches shown on the graph to the left. Let

$$f(x) = \frac{\sqrt{xy}}{x^2+y^2}.$$

(1): Along this path, $y = x > 0$, so

$$f(x,x) = \frac{|x|}{2x^2} = \frac{x}{2x^2} \to \infty \text{ as } x \to 0$$

(2): Along this path, $y = x < 0$, so

$$f(x,x) = \frac{|x|}{2x^2} = \frac{-x}{2x^2} \to -\infty \text{ as } x \to 0$$

So not only is $f(x,y)$ unbounded at the origin, but the limit is path dependent.