Arts & Science 1D06 Quiz #12

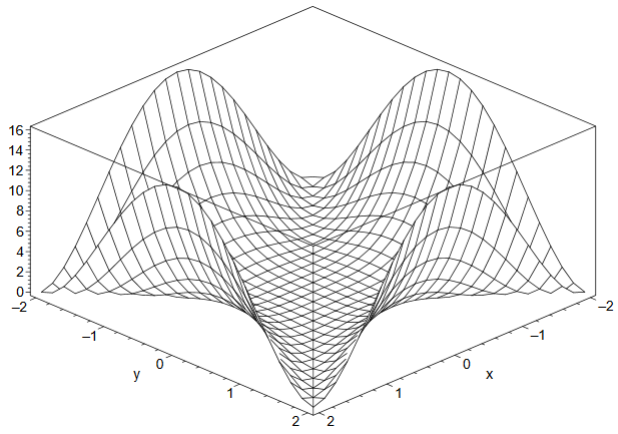
March 30, 2016

Full Name:_____ Student # :_____

TA:_____

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[5 marks] (1.) Which function is graphed below? Explain your answer.



(a) $f(x,y) = (x-y)^2$ (b) $f(x,y) = |xy|$ (c) $f(x,y) = |x| + |y|$ (d) $f(x,y) = (x^2 - y^2)^2$

The function graphed is (d) $f(x,y) = (x^2 - y^2)^2$. One way you can see this is by noting that the graph is 0 along the lines $y = \pm x$. This is only consistent with (d), as $f = 0$ when $x^2 + y^2 = 0$, or $y = \pm x$.

[5 marks] (2.) Explain why the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4 + 3y^4}$$

Along the path $x = 0$:

$$\lim_{(0,y)\to(0,0)} \frac{(0)(y^2)}{(0)^4 + 3y^4} = 0$$

Along the path $y = x$:

$$\lim_{(x,x)\to(0,0)} \frac{(x^2)(x^2)}{x^4 + 3x^4} = \lim_{(x,x)\to(0,0)} \frac{x^4}{4x^4} = \frac{1}{4}$$

Since $0 \neq \frac{1}{4}$, we can conclude that the limit does not exist.