Arts & Science 1D06 Quiz #11

17 March 2016

Full Name: SOLUTIONS Student # :

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Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

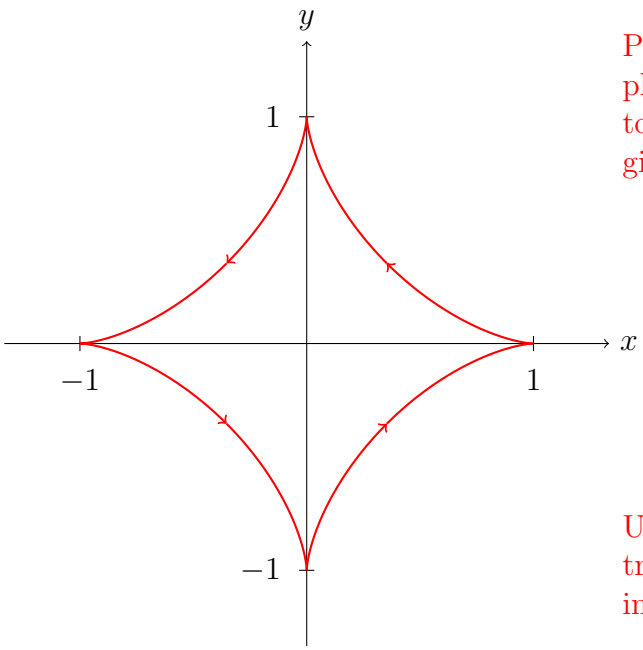
[5 marks]

(1) The astroid is a curve that can be parametrized by the equations

$$\begin{cases} x &= \cos^3(t) \\ y &= \sin^3(t) \end{cases} \qquad 0 \le t \le 2\pi$$

Sketch the astroid, given the above parametrization. Using this parametrization, would a particle travelling along the curve move in the clockwise direction?

[5 marks]



Plotting a couple points (I used multiples of $\pi/4$), we should get the shape to our left, with the direction of travel given by the arrows

Using this parametrization, a particle travelling along the curve would move in the counter-clockwise direction

(2) Consider the curve paramtrized by $x = e^t$, $y = t^2 + t$, where t ranges over all real numbers. Where is the tangent to the curve horizontal?

To solve this question, we'll need to know where $\frac{dy}{dx}$ is zero. This is equivalent to checking that $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$ and is finite. Since $\frac{dx}{dt} = e^t \neq 0$ and is finite for any t, we can just look at $\frac{dy}{dt}$:

$$\frac{dy}{dt} = 2t + 1 = 0 \Leftrightarrow t = -\frac{1}{2}$$

So the tangent to the curve is horizontal when $t = -1/2$, which is when

$$(x, y) = (e^{-1/2}, -1/4)$$