Arts & Science 1D06 Quiz #10

3 March 2016

Full Name: SOLUTIONS Student # :

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Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[4 marks] (1) Solve the differential equation

$$\frac{dy}{dx} = \frac{xe^{x^2}}{y^2}$$

[Note: you do not have to express y as an explicit function of x.]

This is a separable equation, so

$$\begin{aligned} \frac{dy}{dx} &= \frac{xe^{x^2}}{y^2} \\ y^2 dy &= xe^{x^2} dx \\ \frac{1}{3}y^3 &= \frac{1}{2}e^{x^2} + C \\ y^3 &= \frac{3}{2}e^{x^2} + C \end{aligned}$$

[6 marks] (2) Solve the initial value poblem

$$\frac{dy}{dx} = 2xy + 2x - y - 1, \quad y(0) = 1$$

(1) First we'll solve the DE by factoring:

$$\begin{aligned} \frac{dy}{dx} &= (2x-1)(y+1) \\ \frac{dy}{y+1} &= (2x-1)dx \\ \ln|y+1| &= x^2 - x + C \\ y+1 &= Ae^{x^2-x} \\ y &= Ae^{x^2-x} - 1 \end{aligned}$$

(2) Now we'll use the initial condition $y(0) = 1$ to solve for A:

$$\begin{aligned} y(0) = 1 &= Ae^{0^2-0} - 1 \\ 1 &= A - 1 \\ A &= 2 \end{aligned}$$

So the solution to the IVP is

$$y = 2e^{x^2-x} - 1$$