Arts & Science 1D06 Quiz #10SOLUTIONS 26 February 2016

Full Name:______________________________ Student # :______________

TA:______________________________

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[5 marks] (1) Solve this differential equation to get an explicit expression for y.

$$\frac{x}{y}y' = x^3 + 1.$$

This is a separable equation, and we can write it like this:

$$\frac{dy}{y} = \frac{x^3+1}{x}\,dx = \left(x^2 + \frac{1}{x}\right)dx.$$

Integrating both sides gives

$$\ln(y) = \frac{x^3}{3} + \ln(x) + C,$$

where C is a constant. Taking e to the power of both sides yields

$$y = \exp\left(\frac{x^3}{3} + \ln(x) + C\right) = e^C e^{\frac{x^3}{3}} e^{\ln(x)} = Ae^{\frac{x^3}{3}}x.$$

[5 marks] (2) Solve this initial value problem.

$$y' = \sin(x) + y^2\sin(x) + x + y^2x, \qquad y(0) = 0.$$

Though this doesn't initially look separable, we can factor the right-hand side of the Diffy Q in a very cute way:

$$y' = \sin(x)\left(1+y^2\right) + x(1+y^2) = (\sin(x)+x)\,(1+y^2),$$

and this allows us to separate the equation

$$\frac{dy}{1+y^2} = (\sin(x)+x)\,dx,$$

and integrating both sides gives

$$\arctan(y) = -\cos(x) + \frac{x^2}{2} + C \implies y = \tan\left(-\cos(x) + \frac{x^2}{2} + C\right).$$

To solve the initial value problem, I'll plug in $x = 0$, which makes the right side $\tan(\cos(0) + 0 + C) = \tan(C+1)$, which is equal to zero if $C = -1$.