Arts & Science 1D06 Quiz #1

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Full Name: Solutions

Student # :

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Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[4 marks] (1) Let $y = x^2 + 2^x + 2^2$. Compute $\dfrac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 + 2^x + 2^2\right)$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(2^x) + \frac{d}{dx}(2^2)$$

$$= 2x + 2^x \ln 2 + 0$$

$$= 2x + 2^x \ln 2$$

[6 marks] (2) Let g be a differentiable function on all of $\mathbb{R}$ such that $g(\pi/2) = g'(\pi/2) = 1$. If $f(x) = \dfrac{xg(x)}{\sin x}$, compute $f'(\pi/2)$.

- Use quotient & product rules.

$$f'(x) = \frac{(xg(x))' \sin x - (xg(x))(\sin x)'}{(\sin x)^2}$$

$$= \frac{(g(x) + xg'(x))\sin x - xg(x)\cos x}{\sin^2 x}$$

so
$$f'(\pi/2) = \frac{\left(g(\frac{\pi}{2}) + \frac{\pi}{2}g'(\frac{\pi}{2})\right)\sin(\frac{\pi}{2}) - \cancel{\frac{\pi}{2}g(\frac{\pi}{2})\cos(\frac{\pi}{2})} \to 0}{\sin^2(\frac{\pi}{2})} \text{ (since } \cos\tfrac{\pi}{2} = 0)$$

$$= \frac{(1 + \frac{\pi}{2}\cdot 1)\cdot 1}{1^2}$$

$$= 1 + \frac{\pi}{2} = \frac{2+\pi}{2}$$