Comparing the Recovery Models

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April 20, 2005

1 Recovery of treasury

Let us first discuss the RT model, since it is the weakest candidate of the three. The weaknesses of the RT model are:

(1) Coupon bonds can recover more than their par value.

This may occur for example if the defaultable coupon bond has a high default risk, a long time to maturity, and trades close to par. Then the equivalent default-free coupon bond will have a value that is significantly above par.

For Example, we take default intensity $\lambda = 5\%$, default-free interest rate $r = 4\%$ at all times, and the average RT recovery rate $c = 40\%$. Then defaultable coupon bonds of 5 and 10 years to maturity annual coupons are

$$\bar{c}_5 = 7.0967\%$$

and

$$\bar{c}_{10} = 6.9584\%.$$

Default-free coupon bonds with 5 and 10 years to maturity are $c_5 = 113.39, c_{10} = 123.24$. In this case of the 10-year bond with recovery rate $c = 82\%$ or more would result in a payoff $c\bar{C}$ at default of more than 100.

(2)There is an upper bound for market-observed credit spreads.

Consider the yield spread s^y_{RT} of a defaultable zero-coupon bond, which is defined by

$$\bar{B}_{RT}(0,T) = B(0,T)e^{-s^y_{RT}(0,T)T} \tag{1}$$

In RT model, the price of the defaultable zero-coupon bond is given by

$$\bar{B}_{RT}(0,T) = (1-c)\bar{B}(0,T) + cB(0,T) \tag{2}$$

By equation (1) and (2), we can get that

$$e^{-s^y_{RT}(0,T)T} = \frac{\bar{B}_{RT}(0,T)}{B(0,T)} = (1-c)P(0,T) + c > c \tag{3}$$

By taking logarithms and dividing both sides by -T, we can get the expression of yield spread and its upper bound as follows:

$$s^y_{RT}(0,T) = -\frac{1}{T} ln[(1-c)P(0,T) + c] \tag{4}$$

and

$$s^y_{RT}(0,T) < -\frac{1}{T} lnc \tag{5}$$

This upper bound for the credit spread, $-\frac{1}{T}lnc$, is significant, and does not depend on the default intensity. If we take $c = 0.4$, then $-lnc \approx 0.916$. For $T = 10$ it means that $s^y_{RT} < 9.16\%$ whatever the actual default risk of the obligor.

These points highlight a fundamental problem of the RT model: A constant RT recovery rate means different loss severities for bonds of different maturities and different loss severities for bonds of different coupon sizes.

Now let us analyse the par spread of defaultable coupon bonds under RT model.

The price of defaultable coupon bond is:

$$\bar{C}_{RT} = (1-c)\bar{C} + cC = (1-c)\sum_{i=1}^{N} \bar{c}\bar{B}(0,T_i) + c\sum_{i=1}^{N} \bar{B}(0,T_i) + (1-c)\bar{B}(0,T_N) + cB(0,T_N).$$

Set $\bar{C}_{RT} = 1$ to solve for the coupon $\bar{c}$ to get:

$$\bar{c}^{par}_{RT} = \frac{1 - cB(0,T_N) - (1-c)\bar{B}(0,T_N)}{c\sum_{i=1}^{N} B(0,T_i) + (1-c)\sum_{i=1}^{N} \bar{B}(0,T_i)} \tag{6}$$

Then subtracting the default-free par coupon amount from this gives the par coupon spread

s_{RT}^{par} for the RT model:

$$s_{RT}^{par} = \frac{(1-\bar{B}(0,T_N))\sum_{i=1}^{N} B(0,T_i) - (1-B(0,T_N))\sum_{i=1}^{N} \bar{B}(0,T_i)}{\sum_{i=1}^{N} B(0,T_i)(c\sum_{i=1}^{N} B(0,T_i) + (1-c)\sum_{i=1}^{N} \bar{B}(0,T_i))} \tag{7}$$

There is also an upper bound for par spreads. This is reached by letting the default intensity approach infinity or setting $\bar{B}(0,T_i) = 0$ for all $i \leq N$. The upper bound for par spread is:

$$s_{RT}^{par} \leq \frac{1}{c\sum_{i=1}^{N} B(0,T_i)}. \tag{8}$$

In RT model, par spread is bounded from above and the bound is independent of default intensity, which is very much like the properties of the zero-coupon bond yield spreads.

Now we take recovery rate $c = 40\%$, default-free interest rates $r = 4\%$, and with a constant intensity Poisson process with intensity $\lambda = 5\%$, then annual coupons to get the following graph. Please find the figure at Appendix 1.

In the figure 1, the red line stands for upper bound of the zero-coupon spread. Green line stands for par coupon spreads and blue line for zero-coupon spreads. From the graph, we can see that the par spread curve and zero-coupon bond spread are monotonically decreasing. But zero-coupon bond spread curve is a little bit lower than the par spread curve. One can also see that the upper bound for the zero-coupon bond spreads can become a significantly binding restraint.

- For a given par spread curve, the implied intensity can be obtained. For example, if the par spread curve is 3%, i.e. $s_{RT}^{par} = 3\%$, and $r = 4\%, c = 40\%$, then the corresponding term structure of implied default intensities is influenced by the upper bounds on the par spreads. The closer the par spread curve comes to the upper bound, the higher the resulting implied default intensity.

- Coupon differences: Because the recovery in the RT model is specified in terms of all defaultable claims, a different coupon amount will result in different recovery payoff for the bond.

- Behavior for high-yield bonds: The high coupons and default intensities of high-yield bonds exacerbate the problems of RT model. For example, if $s_{RT}^{par} = 7\%$, the par spread curve can not be fitted beyond a maturity of 14 years, and the equivalent four-year default-free bond is worth 125.34.

- Conclusion about RT model:

  The biggest strength of the RT model is computational convenience. If a price for the corresponding default-free payoff is already available, and a model for the zero-recovery case has also already been built, then the RT model does not require anything but forming a weighted average of the default-free and zero-recovery rate. Unfortunately, significant shortcomings is its shape of spread curves and intensities and need some complicated the adjustment for the recoveries above 100%

  Thus, RT is convenient as a quick fix to add some kind of recovery to models that previously only had zero recovery, but otherwise it can not be recommended for credit derivative pricing.

2 Recovery of Market Value (RMV)

As opposed to the RT model, RMV does not impose unrealistic bounds on spreads. By definition, the par coupon of a defaultable coupon bond with default intensity λ, loss quota q and default-free interest rate r must ensure that the price of the defaultable bond is at par after the payment of the coupon. If coupons are paid at intervals of Δt, the par coupon amount is

$$e^{(r+q\lambda)\Delta t} - 1 \tag{9}$$

and the par coupon spread is

$$s_{RMV}^{par} = e^{(r+q\lambda)\Delta t} - e^{r\Delta t} \tag{10}$$

From the equation (9) we can see that the par coupon amount is independent of the maturity of the bond, and there are no upper bounds to the par spreads and zero-coupon bond spreads. This means that the problems of the RT model are avoided. Then we can immediately answer some test questions.

- From equation (10), for constant default intensities, the par spread curves are also constant over maturity.

- For a given par spread, the default intensity curve that corresponds to a given flat par spread curve is a again flat.

- If two defaultable bonds are identical except for the sizes of their respective coupons, they will have different pre-default prices. Thus, they will have different payoffs at default.

For par coupon bonds, the RMV and RP models imply the same payoffs at default. The pre-default market value and the par value of this bond coincide in this case. Therefore we expect the RP model to exhibit similar properties to the RMV model. The par coupon amount in the RP model is:

$$\begin{aligned} \frac{r+\lambda(1-\pi)}{r+\lambda}(e^{(r+\lambda)\Delta t}-1) &\approx \frac{r+\lambda(1-\pi)}{r+\lambda}(1+(r+\lambda)\Delta t-1) \\ &= r\Delta t+(1-\pi)\lambda\Delta t. \end{aligned}$$

The par spread in the RP model is:

$$\begin{aligned} s_{RP}^{par} &= \frac{r+\lambda(1-\pi)}{r+\lambda}(e^{(r+\lambda)\Delta t}-1)-(e^{r\Delta t}-1) \\ &\approx \frac{r+\lambda(1-\pi)}{r+\lambda}((r+\lambda)\Delta t+1-1)-(r\Delta t+1-1) \\ &= \Delta t[r+(1-\pi)\lambda] \end{aligned}$$

This means that for small Δt, the par coupon amount and the par coupon of the RMV model are both approximately $r\Delta t+(1-\pi)\lambda\Delta t$.

This confirms that RMV and RP are equivalent for all securities whose market price is close to the corresponding par value.

The RP and RMV model also have some differences. Differences between RP and RMV occur when the prices of the bonds are far away from par. We can sum up the pros and cons of RMV and RP as follows:

Situation	RMV	RP
Pricing of par bonds close to par	similar to RP	similar to RMV
Pricing of recovery of OTC derivatives	very good	must mimick RMV
Downgraded and distressed debt	recovery assumption must be adjusted	recovery assumption can remain unchanged as long as the market price is above the recovery rates
Differing coupon sizes, same par value	different recovery rates-price difference is the value of the coupon difference under RMV recovery	same recovery rates-price difference is the value of the coupon difference under zero recovery
Pricing formulae	elegant, simply discount with adjusted "defaultalbe" rate	more complicated, need to integrate/sum over all possible times of default
Estimation and calibration	no artificial distortion in the calibration of implied hazard rates from par bond spreads, simple consistent estimation (of product process $q\lambda$) no separate calibration of λ and q possible	no artificial distortion in the calibration of implied hazard rates from par bond spreads, may be able to calibrate implied recovery rate and default intensity
Modelling issues	only need to model dynamics of product process for pricing, OK if used with intensity-based models, cannot be used in firm's value-based	model closer to default payoff definition in CDS documentation, can also be used in models where default is gradually approached
"Story"	reorganisation, renegotiation of debt	bankruptcy proceedings under an authority ensuring strict relative priority

Conclusions about RP and RMV models:

Both RP and RMV have their strengths and weaknesses, and given the uncertainty surrounding the value or distribution of recovery rates, small theoretical differences will not make much difference in many application scenarios.

RMV is mathematically more elegant and is a good model for counterparty risk in OTC derivatives transactions, but in particular for debt trading far below par it has conceptual and modelling problems.

RP seems to do better here. Its setup is closely related to the way historical recovery data is collected.

Therefore, RP wins the race by small margin. RT dropped out early on, and the role of RMV is unclear for crisis scenarios, although it is a good model in other respects. The additional work that RP requires in order to determine the value of the recovery payoffs will have to be done anyway if a CDS is to be priced.

3 Empirical Analysis of Recovery Rates

(1) Market implied recovery rates

In general, recovery rates are difficult to imply from observed market prices. Instead of trying to imply recovery rates from debt prices of the same seniority class, Unal *et al.* (2001) use the price differentials between senior- and junior-rated debt of the same obligor to imply expected recovery rates.

The basic idea is the following:

Let A be underlying value for the "assets in default", the total amount that is paid to creditors. π^s is the recovery payoff. We specify the absolute priority as:

$$\pi^S(A) = \frac{1}{K^S} min K^S, A \tag{11}$$

It means that the the recovery payoff π^s of the senior debt is the whole value of the assets in default A, until the notional K^S of senior debt is fully paid off. The corresponding payoff to junior debt is:

$$\pi^J(A) = \frac{1}{K^J} max A - K^S, 0 \tag{12}$$

and

$$K^J \pi^J + K^S \pi^S = A. \tag{13}$$

Stochastic recovery is modelled by modelling A as a random variable. Suppose the conditional distribution of A be a beta distribution with fixed variance but unknown mean μ with density function $f(\mu, .)$. Then the expected recovery payoffs are:

$$\pi^{Se}(\mu) = \int \pi^S(a) f_A(\mu, a) da \tag{14}$$

and

$$\pi^{Je}(\mu) = \int \pi^J(a) f_A(\mu, a) da \tag{15}$$

The model prices for the defaultable bonds are given by substituting these expected recovery values in the corresponding bond pricing formula. Setting model prices equal to market prices yields two equations for two unknown: mean asset-in-default value μ and default hazard rate h.

(2)Historical recovery rates

When market implied recovery rates are not available, historical recovery rates can give a valuable benchmark for appropriate values. Table 1 shows mean, median and standard deviation of the recovery rates for different debt classes over the years 1981-2000.

Table 1.

Seniority/security	Median	Average	SD	1st Quartile	3rd Quartile
Senior/secured bls	72.0	64.0	24.4	45.3	85.0
Senior/unsecured bls	45.0	49.0	28.4	25.0	75.8
Senior/secured bonds	53.8	52.6	24.6	34.8	68.6
Senior/unsecured bonds	44.0	46.9	28.0	25.0	66.8
Senior/subordinated bonds	29.0	34.7	24.6	15.1	50.0
Subordinated bonds	28.5	31.6	21.2	15.0	44.1
Jenior/subordinated bonds	15.1	22.5	18.7	11.3	33.0
Preferred stock	11.1	18.1	17.2	6.4	24.9

Recovery rates show extremely high variability across different default events. Some empirical indicators have been found that explain some of this variability, but much uncertainty remains. From Table 1 we can see that, the large SDs are an indicator for the uncertainty surrounding real-world recovery rates.

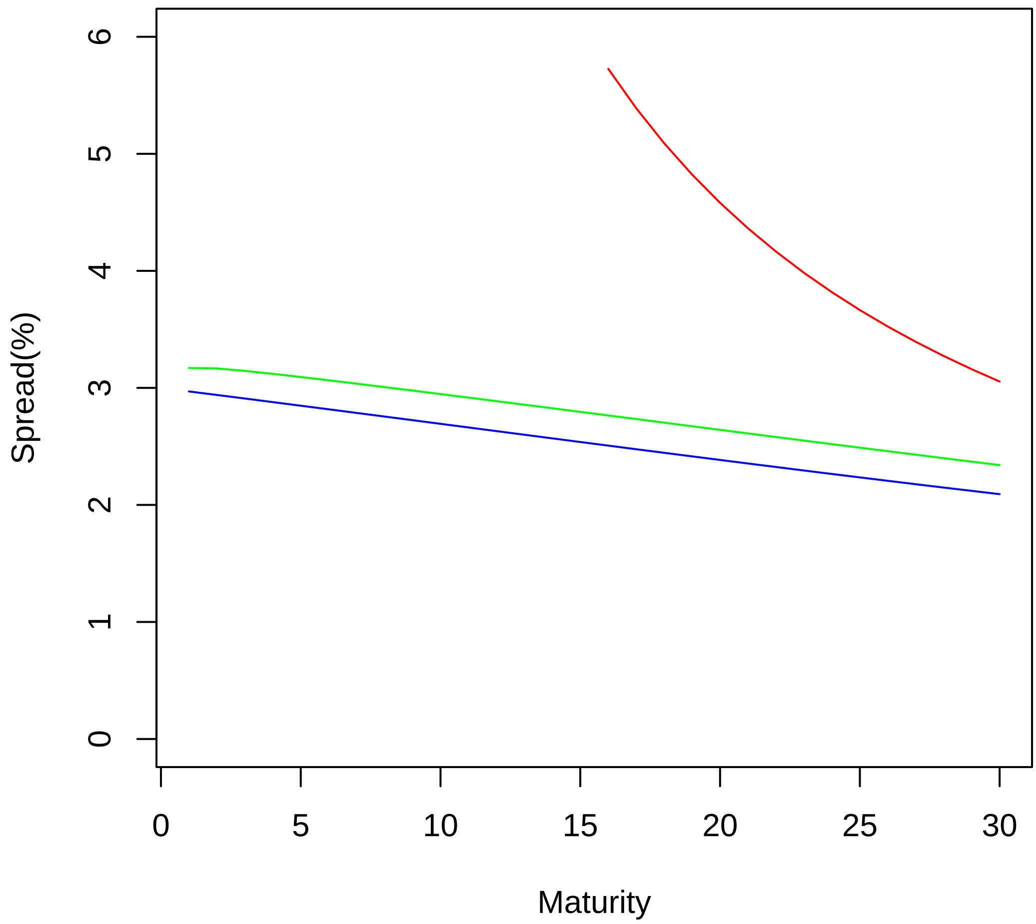
Table 2. Recovery rates by type of collateral. Collateral codes: all assets and current assets=5; most assets=4; secured transactions, real estate, PP&E, oil and gas reserves and equipement=3; capital stock of operating units, intellectual property and intercompany debt=2; second lien=1; unsecured=0

Collateral	Recovery	SD	95% Conf.	Count
(1-5)	79.6	26.7	35.5	327
(2-5)	80.6	25.9	37.8	312
(3-5)	82.9	25.3	41.2	254
(4-5)	86.3	23.7	47.2	174
(5)	89.8	19.8	57.2	157

The principle findings for this method to get the above Table 2 are:

- Higher seniority $\Rightarrow$ higher recovery at default.
- The level of subordination and security are strong influences.
- Better rating $\Rightarrow$ higher recovery.
- Default losses are correlated with the leverage of the defaulting firm with higher leverage implying higher losses.
- The relationship between loss and leverage is stronger in business cycle downturns yielding very low recoveries for highly leveraged firms in recessions.
- Secure debt is less sensitive to the general state of the economy than unsecured debt.

Appendix 1: Spreads in the recovery of treasury (RT) model.



Appendix 2: Code for the figure of the spread. N=30;c=0.4;r=0.04;lambda=0.05

BbarTN=parspread=upperbound=array(dim=N) BTN=T=array(dim=N) sumBTi=sumBbarTi=arra

for (i in 1:N) T[i]=i

for (i in 1:15) upperbound[i]=NA

for (i in 16:N) upperbound[i]=-1/T[i]*log(c, base=exp(1)) upperbound

yieldspread=-(1/T)*log((1-c)*exp(-lambda*T)+c, base=exp(1)) yieldspread

for (i in 1:N) BbarTN[i]=exp(-(r+lambda)*i) BTN[i]=exp(-r*i)

sumBTi[1]=BTN[1] sumBbarTi[1]=BbarTN[1] for (i in 2:30) t1=exp(-r*i) t2=exp(-(r+lambda)*i) sumBTi[i]=sumBTi[i-1]+t1 sumBbarTi[i]=sumBbarTi[i]+t2

for (i in 1:30)

parspread[i]=((1-BbarTN[i])*sumBTi[i]-(1-BTN[i])*sumBbarTi[i])/(sumBTi[i]*(c*sumBTi[i]+(1-c)*sumBbarTi[i])) parspread

plot(T, type="l", upperbound*100, xlab="Maturity", ylab="Spread(ylim=c(0,6), col="red")

lines(T,parspread*100,col="green") lines(T,yieldspread*100,col="blue")