Business V703 Financial Modeling Valuation

Lecture 8 - Introduction to Game Theory

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Monopoly versus competition

- The model so far has assumed a monopoly right to invest in a project.
- The option value of waiting produced wider price ranges for investment/abandonment.
- This leads to a more conservative attitude than predicted by a NPV approach.
- With competition, the opportunity value of waiting should decrease.
- How can we incorporate the effect of competition ?
- How does it affect our results ?

Prerequisites for a theory of competition

- We need a framework that incorporates the actions of competitors, as well as their response to our actions.
- The theory should also deal with the fact the competitors also try to predict our actions as well as our response to their actions.
- Finally, it should take into account the effect of uncertainty on all the participants.
- Prototype problem: fugitive crossing 3 distinct bridges.
- The most comprehensive theoretical framework with all these prerequisites is Game Theory.

Philosophical examples of strategic thinking gone bad

- In the Laches, Socrates worries that a soldier can be led to run away from a battle if he considers the thought process of other soldiers:

  You're generally pretty safe if that's the way you look when you're in action; it's the man whose one idea is to get away that the other fellow goes for.

- In modern times, the character Yossarian in Catch 22 (Joseph Heller) does not want to be one of the last to die in in WWII:

  - From now on I am thinking only of me.
  - But Yossarian, suppose everyone felt that way.
  - Then I would be a damned fool to feel any other way, wouldnt I?

- Such considerations also inspired contractarian political philosophy, as in Hobbes's Leviathan:

  Life in a state of nature is solitary, poor, nasty, brutish and short.

Literary and historical precedents for Game Theory

- In Henry V, Act IV, scene 6,
  on the eve of the Battle of Agincourt, the English king declares:

  But, hark! what new alarum is this same?
  The French have reinforced their scatter'd men:
  Then every soldier kill his prisoners
  Give the word through.

- One century later, Hernan Cortez burned the ships in which he and his troops arrived in Mexico, therefore preventing his forces to act like Yossarian or the socratic soldier.
- Modern day armies threaten to kill or severely punish deserters.

Elements of a Game

- Players are assumed to behave rationally. That is, they make decisions in order to maximize the utility of their payoffs.
- Games can be played in either sequential or simultaneous timing (often involving mixtures of the two).
- At each stage of a game, players might have access to perfect of imperfect information.
- The collection of a player's decisions in all possible stages of a game is called a strategy.
- Games are generally analyzed either in strategic form (matrices) or sequential form (trees).

The Prisoner's Dilemma

- Suspects A and B are arrested and accused of collaborating in robbing a bank. Prosecutors do not have enough evidence to convict them of bank robbery, but can get them convicted for stealing the runaway car.
- The police tells each of them that:
  1. if one confesses the crime while the other refuses to cooperate, he is set free and the other goes to jail for 10 years;
  2. if both confess, then they both go to jail for 5 years;
  3. if both refuse to cooperate, then they both go to jail for 1 year.

Strategic Analysis

- Analysis in strategic form is often best suited for games with simultaneous moves.
- The strategic table for the PD is as follows:

		B: Confess	B: Hold
A	Confess	**(-5,-5)**	(0,-10)
	Hold	(-10,0)	(-1,-1)

- In each pair (a, b), the first number corresponds to the payoff for player A while the second correspond to the payoff for player B.

Strictly dominated strategies

- Analyzing the game from the point of view of player A, we see that the second row is strictly dominated by the first, and should be eliminated from the possible solutions.
- For the remaining first row, B will choose the top-left cell, leading to simultaneous confession as the solution.
- Alternatively, from the point of view of B, the second column is strictly dominated by the first, and should be eliminated.
- For the remaining first column, A will choose the top-left cell as well.
- The strategies (Confess,Confess) correspond to Nash equilibrium (NE): the best response from each player given the choice of the others.
- It is generally agreed that being a NE is a necessary condition for a solution of a game.

Weakly dominated strategies

- Consider the following example (Krepps 1990):

	B1	B2
A1	**(10,10)**	(0,0)
A2	(0,0)	**(1,1)**

Here there are no strictly dominant strategies, and both (A1,B1) and (A2,B2) are NE. While clearly (A1,B1) seems preferable to (A2,B2), we cannot arrive at this conclusion based on the rules presented so far.

- Alternatively, consider the example (Krepps 1990):

	B1	B2
A1	(10,0)	**(5,2)**
A2	(10,1)	(2,0)

Here both (A1,B2) and (A2,B1) are NE. We also have that the second row is weakly dominated by the first, leading us to the solution (A1,B2).

Examples (continued)

- Modifying the previous example slightly leads to (Kreps 1990):

	B1	B2
A1	(10,0)	**(5,2)**
A2	(10,11)	(2,0)

  Here both (A1,B2) and (A2,B1) are NE and the second row is still weakly dominated by the first. We are then led to the solution (A1,B2), while (A2,B1) seems preferable.

- These added rules for selecting a solution (other than strict dominance and NE) are called refinements.

A sequential version of the PD

- Back to the prisoners dilemma, it might be argued that the preferred outcome (Hold,Hold) (which is not a NE), can arise as the solution to the game if the prisoners are allowed to communicate and agree to follow it.
- In this case, despite agreeing to follow (Hold,Hold), the prisoners will still choose mutual confession (talk is cheap !).
- A possible modification would be a sequential version of the PD, where one prisoner decides what to do only after observing what the other prisoner does.
- This can then be solved by Zermelo's algorithm (backward induction) on a tree, in very much the same way that we did option price.
- Depressingly, the solution to this modified version of the PD is still mutual confession.
- Let us do this on the blackboard !

Subgame perfection

- Consider now the game proposed in Ross (2005) (to be described in the blackboard!).
- In strategic form, we have:

		B			
		DD	DU	UD	UU
A	DD	(3,3)	(3,3)	(0,5)	(0,5)
	DU	(3,3)	(3,3)	**(0,5)**	(0,5)
	UD	(-1,0)	(4,5)	(-1,0)	(4,5)
	UU	(-1,0)	(5,-1)	(-1,0)	(5,-1)

  where, for instance, the strategy (UD, UD) means that A goes 'up' on node 1 and 'down' on node 4, and B goes 'up' on node 2 and 'down' on node 3.
- Here the NE are (DD, UD) and (DU, UD).
- However, (DD, UD) is not an equilibrium for the subgame started at node 4.
- On the other hand, the sequential analysis for this game selects (DU, UD) as subgame perfect equilibrium (SPE).