Business V703 Financial Modeling Valuation

Lecture 7: Investment, suspension and abandonment

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The option to mothball

- Suppose that, instead of completely abandoning the project, we have the option to mothball it (i.e temporarily suspend it), while prices remain unfavorable.
- As before, let I be the sunk cost for an initial investment and C be the rate of operating costs in the active phase.
- Additionally, let E_M be the sunk cost for mothballing an active project and C_M be the rate of maintenance costs for a mothballed project.
- Finally, let I_R be the sunk cost necessary to reactivate a mothballed project and E_S be the scrapping sunk cost (which could be negative) for permanently abandoning it.
- Mothballing in only viable if $C_M < C$ and/or $I_R < I$. We assume that both are true.

Price thresholds

- What triggers the decisions to invest, mothball, reactivate or scrap ?
- Initial investment threshold: invest when $P \geq P_H$.
- Mothballing threshold: suspend operations when $P \leq P_M$.
- Reactivation threshold: reactivate when $P \geq P_R$.
- Scrapping threshold: scrap when $P \leq P_S$.
- We expect that $P_S < P_M < P_R < P_H$.

Project values and options

- As before, we denote the value of an idle project by F^0 and the value of an active project by F^1.
- We further introduce F^M as the value of a mothballed project.
- Then

$$\begin{aligned} F^0 &= \text{option to invest at cost } I \\ F^1 &= \text{cash flow } + \text{option to mothball at cost } E_M \\ F^M &= \text{cash flow } + \text{option to reactivate at cost } I_R \\ &\quad + \text{option to scrap at cost } E_S \end{aligned}$$

- When the underlying output flow rate is P, the present value for the cash flows of an active project is given, as before, by $\frac{P}{\delta} - \frac{C}{r}$.
- Similarly, the present value for the cash flows of a mothballed project is $-\frac{C_M}{r}$.

Grid Values

- As before, the underlying values for P_i are given by

$$P_i = P_{\bar{m}} e^{(\bar{m}-i)\sigma\sqrt{\Delta t}}, \qquad i = 1, \dots m,$$

  where $\bar{m}$ denotes a middle row. We then complement this with $P_{m+1} = 0$.
- Next, denoting by F_{ij}^k the value for the project in phase $k = 0, 1, M$ at time t_j when the underlying output rate is P_i, we obtain its value on the grid using the recursion formula

$$F_{ij}^k = \max\{\text{continuation value, possible exercise values}\}.$$

Boundary values

- At the bottom of the grid we have, for all $j = 1, \ldots, n+1$:

$$\begin{aligned} F^0_{m+1,j} &= 0 \\ F^1_{m+1,j} &= -(E_S + E_M) \\ F^M_{m+1,j} &= -E_S \end{aligned}$$

- Similarly, at the top of the grid we should have, for all $j = 1. \ldots, n+1$:

$$\begin{aligned} F^0_{1j} &= \frac{P_1}{\delta} - \frac{C}{r} - I \\ F^1_{1j} &= \frac{P_1}{\delta} - \frac{C}{r} \\ F^M_{1j} &= \frac{P_1}{\delta} - \frac{C}{r} - I_R \end{aligned}$$

Boundary values (continued)

- Finally, at the final time we have, for $i = 1, \ldots m+1$:

$$
\begin{aligned}
F^0_{i,n+1} &= \max\left[0, \left(\frac{P_i}{\delta} - \frac{C}{r}\right) - I\right] \\
F^1_{i,n+1} &= \max\left[\frac{P_i}{\delta} - \frac{C}{r}, -\frac{C_M}{r} - E_M, -E_S - E_M\right] \\
F^M_{i,n+1} &= \max\left[-\frac{C_M}{r}, \left(\frac{P_i}{\delta} - \frac{C}{r}\right) - I_R, -E_S\right]
\end{aligned}
$$

Idle phase

- In the idle phase, the only possibilities are to remain idle or switch to an active project.
- This gives the following continuation and exercise values:

$$\begin{aligned}
\text{cont}_{ij}^{0} &= \frac{qF^{0}_{i-1,j+1} + (1-q)F^{0}_{i+1,j+1}}{e^{r\Delta t}} \\
\text{exer}_{ij}^{0} &= -I + \left(\frac{P_i}{\delta} - \frac{C}{r}\right) \\
&+ \frac{q(F^{1}_{i-1,j+1} - \frac{P_{i-1}}{\delta} + \frac{C}{r}) + (1-q)(F^{1}_{i+1,j+1} - \frac{P_{i+1}}{\delta} + \frac{C}{r})}{e^{r\Delta t}}
\end{aligned}$$

Mothballed phase

- In the mothballed phase, the possibilities are to remain mothballed, switch to an active project or abandon it permanently.
- This gives the following continuation and exercise values:

$$\begin{aligned}
\text{cont}_{ij}^{M} &= -\frac{C_M}{r} + \frac{q(F^M_{i-1,j+1} + \frac{C_M}{r}) + (1-q)(F^M_{i+1,j+1} + \frac{C_M}{r})}{e^{r\Delta t}} \\
\text{exer}_{ij}^{M,1} &= -I_R + \left(\frac{P_i}{\delta} - \frac{C}{r}\right) \\
&+ \frac{q(F^1_{i-1,j+1} - \frac{P_{i-1}}{\delta} + \frac{C}{r}) + (1-q)(F^1_{i+1,j+1} - \frac{P_{i+1}}{\delta} + \frac{C}{r})}{e^{r\Delta t}} \\
\text{exer}_{ij}^{M,0} &= -E_S + \frac{qF^0_{i-1,j+1} + (1-q)F^0_{i+1,j+1}}{e^{r\Delta t}}
\end{aligned}$$

Active phase

- In the active phase, the possibilities are to remain active, suspend or abandon the project.
- This gives the following continuation and exercise values:

$$
\begin{aligned}
\text{cont}_{ij}^{1} &= \left(\frac{P_i}{\delta} - \frac{C}{r}\right) \\
&= \frac{q(F^1_{i-1,j+1} - \frac{P_{i-1}}{\delta} + \frac{C}{r}) + (1-q)(F^1_{i+1,j+1} - \frac{P_{i+1}}{\delta} + \frac{C}{r})}{e^{r\Delta t}} \\
\text{exer}_{ij}^{1,0} &= -E_S - E_M \frac{qF^0_{i-1,j+1} + (1-q)F^0_{i+1,j+1}}{e^{r\Delta t}} \\
\text{exer}_{ij}^{1,M} &= -E_M - \frac{C_M}{r} \\
&+ \frac{q(F^M_{i-1,j+1} + \frac{C_M}{r}) + (1-q)(F^M_{i+1,j+1} + \frac{C_M}{r})}{e^{r\Delta t}}
\end{aligned}
$$

Thresholds

- As before, the entry threshold is the price P_H where $F^0 = F^1 - I$.
- Similarly, the mothballing threshold P_M is the point where $F^1 = F^M - E_M$.
- Analogously, the reactivation threshold P_R is the price for which $F^M = F^1 - I_R$.
- Finally, the scrapping threshold is the point where $F^M = F^0 - E_S$.

Example

- Consider the parameters:

$$\begin{aligned} T &= 30, \quad I = 2, \quad E_M = 0, \quad E_S = 0 \\ C &= 1, \quad \sigma = 0.2, \quad r = 0.05, \quad \delta = 0.05 \end{aligned}$$

- Using this values, we consider the following scenarios:

$$\begin{aligned} C_M &= 0.01, 0.05 \\ I_R &= 0.2, 0.4, 0.6, 0.8, 1 \end{aligned}$$

- We should now run our algorithm and find our own thresholds!