Business V703 Financial Modeling Valuation

Lecture 6: Entry and exit strategies

M. R. Grasselli

Mathematics and Statistics
McMaster University

February 8, 2006

Review of the investment rule

- We have assumed until now that the underlying project has a positive value given by

$$dV = \alpha V_t dt + \sigma V_t dz_t \tag{1}$$

- We have determined that when $V_t < V_t^*$ (that is, before an investment decision is made on the project) the potential investor holds an option with value given by $F(V_t, t) > (V_t - I)$.
- Once the project value hits the threshold V_t^*, an investment decision is made and the option value becomes $F(V_t, t) = V_t - I$ thereafter.
- That is, there are no decisions to be made after the initial investment, since the project value V_t remains positive in perpetuity.
- All we have to do is sit back and collect the returns indefinitely !

Output versus Operating Costs

- The framework of the previous slide ignores the possibility of negative cash flows arising from the active project, for instance, when operating costs exceed the revenue.
- Once this happens, we have to consider the decision to abandon the project.
- We therefore depart from the model (1) for the project value, and instead focus on two other underlying variables: the (random) output cash flow rate P_t and the (fixed) operating cost rate C.
- Given P_t and C, the cash flow obtained during a period Δt is therefore approximately equal to

$$(P_t - C)\Delta t$$

Embedded options

- Modeling P_t as the underlying variable has the effect of turning the project value itself into a derivative, that is, a process whose value depends on P_t.
- The decisions to invest or abandon the project are then viewed as options which are embedded in the project value itself.
- We then divide the overall project value into several different functions $F^0(P,t), F^1(P,t), \ldots$ depending on which options are available for a given level of P.

Zero Operating Cost

- For instance, the case considered in the previous lecture corresponds to $C = 0$ and $P_t = \delta V_t$, where V_t is given by (1).
- Denote $P_t^* = \delta V_t^*$, so that the investment decision can be translated in terms of P, instead of V.
- Then the region $P < P^*$ (which is equivalent to $V < V^*$) corresponds to an idle phase for the project, where its value is given by
$$F^0(P, t) = F(V, t) = F(P/\delta, t).$$
- Similarly, the region $P > P^*$ (which is equivalent to $V > V^*$) corresponds to the active phase for the project, where its value is simply
$$F^1(P, t) = V_t - I = P_t/\delta - I.$$

Exit and Entry Costs

- Consider a project with an output rate P_t governed by

$$dP_t = \alpha P_t dt + \sigma P_t dz_t \tag{2}$$

- Assume a non-zero operating cost rate C.
- Suppose that, whenever the project is idle, the cost to activated it is equal to $I > 0$.
- Moreover, suppose that, whenever the project is active, it can be shut down at a cost equal to E.
- We could have E negative (corresponding to a scrap value), as long as $I + E > 0$ (otherwise we have a money making machine!).

Idle and active projects

- We then divide the project values into the functions $F^0(P,t)$, corresponding to the idle phase, and $F^1(P,t)$, corresponding to the active phase.
- That is,

$$\begin{aligned} F^0(P,t) &= \text{option to invest for a cost } I \\ F^1(P,t) &= \text{cash flow} + \text{option to abandon at a cost } E. \end{aligned}$$

- To value these options, we again appeal to arbitrage arguments.
- For this, as always in this course, we will assume that there exits a financial asset X_t which is perfectly correlated with P_t, that is,

$$dX_t = \mu X_t dt + \sigma X_t dz_t, \tag{3}$$

where $\mu - \alpha = \delta$.

Cash Flow Values

- Before we calculate the option values, it is instructive to calculate the present value of the cash flows if the project is kept active in perpetuity.
- Since C is deterministic, the present value of the future operating cost is given by

$$\int_0^\infty Ce^{-rt}dt = \frac{C}{r}.$$

- On the other hand, for the stochastic output flow rate P_t given by (2) with current value P_0, which is assumed to be correlated with a financial asset X_t given by (3), the present value of future cash flows is

$$\int_0^\infty E[P_t]e^{-\mu t}dt = \int_0^\infty P_0 e^{\alpha t}e^{-\mu t}dt = \frac{P_0}{\mu - \alpha} = \frac{P_0}{\delta}.$$

- Compared this with the case of a project with value given by (1) and paying proportional dividends at a rate D (for which the present value of future cash flows is simply V_0).

Grid Values

- We start by determining the underlying values for $P(i)$ according to

$$P_i = P_{\bar{m}} e^{(\bar{m}-i)\sigma\sqrt{\Delta t}}, \qquad i = 1, \ldots m,$$

where $\bar{m}$ denotes a middle row.

- As before, we complement this by setting $P_{m+1} = 0$.
- Next, denoting by F_{ij}^k the value for the project in phase k at time t_j when the underlying output rate is P_i, we obtain its value on the grid using the recursion formula

$$F_{ij}^k = \max\{\text{continuation value, exercise value}\}.$$

Idle phase

- In the idle phase, we have to choose between remaining idle or switching to an active project.
- In other words, we take the maximum between the following continuation and exercise values:

$$
\begin{aligned}
\text{cont}_{ij}^{0} &= \frac{qF_{i-1,j+1}^{0} + (1-q)F_{i+1,j+1}^{0}}{e^{r\Delta t}} \\
\text{exer}_{ij}^{0} &= \left(\frac{P_i}{\delta} - \frac{C}{r}\right) - I \\
&+ \frac{q(F_{i-1,j+1}^{1} - \frac{P_{i-1}}{\delta} + \frac{C}{r}) + (1-q)(F_{i+1,j+1}^{1} - \frac{P_{i+1}}{\delta} + \frac{C}{r})}{e^{r\Delta t}}
\end{aligned}
$$

Active phase

- In the active phase, we have to choose between remaining active or switching to an idle project.
- In other words, we take the maximum between the following continuation and exercise values:

$$\begin{aligned}
\text{cont}_{ij}^1 &= \frac{P_i}{\delta} - \frac{C}{r} \\
&+ \frac{q(F_{i-1,j+1}^1 - \frac{P_{i-1}}{\delta} + \frac{C}{r}) + (1-q)(F_{i+1,j+1}^1 - \frac{P_{i+1}}{\delta} + \frac{C}{r})}{e^{r\Delta t}}
\end{aligned}$$

$$\text{exer}_{ij}^1 = \frac{qF_{i-1,j+1}^0 + (1-q)F_{i+1,j+1}^0)}{e^{r\Delta t}} - E$$

Boundary values

- For the formulas above to work, we need to specify the values of the options at the boundaries of the grid.
- At the bottom of the grid it (where $P_{m+1} = 0$), we have

$$\begin{aligned} F^0_{m+1,j} &= 0 \\ F^1_{m+1,j} &= -E, \qquad j = 1, \dots, n+1. \end{aligned}$$

- At the top of the grid (where P is at its maximum), we have

$$\begin{aligned} F^1_{1j} &= \frac{P_1}{\delta} - \frac{C}{r} \\ F^0_{1j} &= \frac{P_1}{\delta} - \frac{C}{\delta} - I, \quad j = 1, \dots, n+1. \end{aligned}$$

- Finally, at the final time, we have

$$\begin{aligned} F^0_{i,n+1} &= \max\left[\left(\frac{P_i}{\delta} - \frac{C}{r}\right) - I, 0\right] \\ F^1_{i,n+1} &= \max\left[\frac{P_i}{\delta} - \frac{C}{r}, -E\right], \quad i = 1, \dots m+1 \end{aligned}$$

Thresholds

- As before, for each fixed $t = t_j$ we can obtain a graph of the project values F_{ij}^0 and F_{ij}^1 as functions of the underlying output flow rate P_i.
- We then obtain that the decision to turn an idle project into an active one is determined by an entry threshold $P_{t_j}^H$ obtained when the graph of F_{ij}^0 touches the graph of $(F_{ij}^1 - I)$.
- SImilarly, the decision to shut down an active project is triggered by an exit threshold $P_{t_j}^L$ obtained when the graph of F_{ij}^1 touches the graph of $(F_{ij}^0 - E)$.

Stationarity and Myopic Decisions

- We can again observe that as we move away from the maturity time T, the entry and exit thresholds approach constant values $P^L < P^H$.
- Moreover, we can compare this values with the entry and exit threshold that would be dictated by Marshallian analysis.
- According to NPV, we should invest in the project whenever $\frac{P}{\delta} - \frac{C}{r} > I$.
- Similarly, NPV tells us that we should abandon the project whenever $\frac{P}{\delta} - \frac{C}{r} < -E$.
- However, we can verify from numerical experiments that

$$\frac{P^L}{\delta} < \left(\frac{C}{r} - E\right) < \left(\frac{C}{r} + I\right) < \frac{P^H}{\delta}.$$

Example: copper industry

- Let us now consider the practical example where the project consists of a copper production facility.
- Denote by P the output flow rate obtained from selling 10 million pounds of copper per year.
- Expressing time in years and money in millions of dollars, let us take the parameters:

$$
\begin{aligned}
T &= 30, \quad I = 20, \quad E = 2, \\
C &= 8, \quad \delta = 0.04, \quad r = 0.04, \quad \sigma = 0.2
\end{aligned}
$$

- Using this values, we obtain the Marshallian thresholds

$$
\begin{aligned}
\delta\left(\frac{C}{r} + I\right) &= 8.8 \\
\delta\left(\frac{C}{r} - E\right) &= 7.92
\end{aligned}
$$

- We should now run our algorithm and find our own thresholds!