Business V703 Financial Modeling Valuation

Lecture 11 - Games and Options

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Combining options and games

- One now move to a systematic application of both real options and game theory in strategic decisions.
- The essential idea can be summarized in two rules:
  1. Outcomes of a given game that involve a "wait–and–see" strategy should be calculated by option value arguments.
  2. Once the NE for a given game is found on a decision node, its value becomes the pay-off for an option at that node.
- In this way, option valuation and game theoretical equilibrium become dynamically related in a decision tree.
- In what follows, we denote the NE solution for a given game in bold face within the matrix of outcomes.
- For convenience of notation we will round all number to the nearest integer.

One Stage Strategic Investment

- As a first example, consider two symmetric firms contemplating a total investment $I = 80$ on a project with $V_0 = 100$ and equal probabilities to move up to $V^u = 180$ and down to $V^d = 60$.
- This gives the risk–neutral probability $q = \frac{e^r - d}{u - d} = 0.4$.
- Therefore, using this to calculate the option value for the "wait–and–see" strategy gives the following matrix of outcomes for this game:

		B: Invest	B: Wait
A	Invest	**(10,10)**	(20,0)
	Wait	(0,20)	(18.5,18.5)

Two–stage monopolistic R&D

- Suppose now that for an R&D cost I_0 at time t_0 a firm can develop a technology that can be commercialized at a later time t_1 at a follow-on cost $I_1 = 80$.
- To use the results from our previous example, suppose that the commercial value for the technology at time t_1 is either $V^u = 180$ with probability $p = 0.5$ or $V^d = 60$ with probability $1 - p = 0.5$.
- From our previous calculation, we know that the current value for this follow–on investment opportunity is $c_0 = 37.0356$.
- Since this option is effectively acquired by I_0, we see that the R&D investment is recommended whenever $I_0 < 37.0356$.
- For instance, if $I_0 = 30$, the option analyzes recommends the R&D project, even though its NPV is negative:

$$NPV = -I_0 + [(pV^u + (1-p)V^d) - e^{-r}I_1] = -4.$$

Two–stage competitive R&D

- Suppose now that firm A is the only firm facing the R&D investment at cost $I_0 = -30$ at time t_0, whereas at time t_1 the firms can equally share the follow–on cost $I_1 = 80$.
- We will assume that the technology resulting from the R&D investment is either proprietary, in which case firm A benefits from a larger fraction of the total market value than firm B, or shared, in which case both firms can capture equal fractions of the total market.
- Moreover, we will consider the cases where the competitive reaction is either contrarian or reciprocating.
- In all cases, we will assume that the market value continues to evolve from time t_1 to time t_2 following the same dynamics.
- That is, at time t_2 the possible market values in these two–period tree are

$$V^{uu} = 324, \quad V^{ud} = 108, \quad V^{dd} = 36.$$

Proprietary R&D with contrarian competition

- Let the market share of firm A after the R&D phase be $s = 2/3$ and assume that B takes a contrarian attitude, therefore preserving the total market value (e.g capacity war).
- If demand is high at time t_1 ($V^u = 180$), we have:

		B (follower)	
		Invest	Wait
A (leader)	Invest	**(80,20)**	(100,0)
	Wait	(0,100)	(81,25)

- If demands is low at time t_1 ($V^d = 60$), we have:

		B (follower)	
		Invest	Wait
A (leader)	Invest	(0,-20)	(-20,0)
	Wait	(0,-20)	**(10,0)**

- Then
  $c_A = -I_0 + e^{-r}[q \times 80 + (1-q) \times 10] = -30 + 35 = 5 > 0,$
- whereas $c_B = e^{-r}[q \times 20 + (1-q) \times 0] = 7$
- Therefore the R&D investment is recommended for A.

Proprietary R&D with reciprocating competition

- Suppose that $s = 2/3$ as before, but due to reciprocating actions by B (e.g price war), the total market value drops to $3/4$ of its monopolistic level.
- If demand is high at time t_1 ($V^u = 180$), we have:

A (leader) \ B (follower)	Invest	Wait
Invest	**(50,5)**	(100,0)
Wait	(0,100)	(61,15)

- If demands is low at time t_1 ($V^d = 60$), we have:

A (leader) \ B (follower)	Invest	Wait
Invest	(-10,-25)	(-20,0)
Wait	(0,-20)	**(10,0)**

- Then $c_A = -I_0 + e^{-r}[q \times 50 + (1-q) \times 10] = -6 < 0$
- whereas $c_B = e^{-r}[q \times 5 + (1-q) \times 0] = 2$
- Therefore the R&D investment is **not** recommended for A.

Shared R&D with contrarian competition

- Let $s = 1/2$ and assume that the total market value under competition remains the same.
- If demand is high at time t_1 ($V^u = 180$), we have:

		B (follower) Invest	B (follower) Wait
A (leader)	Invest	**(50,50)**	(100,0)
	Wait	(0,100)	(53,53)

- If demands is low at time t_1 ($V^d = 60$), we have:

		B (follower) Invest	B (follower) Wait
A (leader)	Invest	(-10,-10)	(-20,0)
	Wait	(0,-20)	**(5,5)**

- Then $c_A = -I_0 + e^{-r}[q \times 50 + (1-q) \times 5] = -9 < 0$
- whereas $c_B = e^{-r}[q \times 5 + (1-q) \times 0] = 21$
- Therefore the R&D investment is **not** recommended for A.

Shared R&D with reciprocating competition

- Finally, let $s = 1/2$ and assume that the total market value increases to 5/4 of the monopolistic level (due to reciprocating benefits).
- If demand is high at time t_1 ($V^u = 180$), we have:

B (follower)

		Invest	Wait
A (leader)	Invest	**(73,73)**	(100,0)
	Wait	(0,100)	(75,75)

- If demands is low at time t_1 ($V^d = 60$), we have:

B (follower)

		Invest	Wait
A (leader)	Invest	(-3,-3)	(-20,0)
	Wait	(0,-20)	**(10,10)**

- Then $c_A = -I_0 + e^{-r}[q \times 73 + (1-q) \times 10] = 3 > 0$
- whereas $c_B = e^{-r}[q \times 73 + (1-q) \times 10] = 33$
- Therefore the R&D investment is recommended for A.

Summary for R&D under competition in the second stage

- Taking the point of view of A, we can summarize the effect of an R&D investment carried by a pioneer A followed by later competition with B as follows:

		B (follower)	
		Contrarian	Reciprocating
A (leader)	Proprietary	$+$	$-$
	Shared	$-$	$+$

Competition in the RD phase

- Suppose that an R&D investment of $I_0 = 30$ can be either made immediately at time $t = 0$ or deferred until time $t = 1$.
- Assume that the underlying project values are $V_0 = 100$ at time $t = 0$, then either $V^u = 180$ or $V^d = 60$ at time $t = 1$ and $V^{uu} = 324$, $V^{ud} = 108$ and $V^{dd} = 36$ at time $t = 2$.
- As before, this gives a risk–neutral probability $q = \frac{e^r - d}{u - d} = 0.4$.
- Now assume that the firms can explore further commercialization at a cost $I = 80$ only after investing in R&D.
- Consider further that competition is reciprocating in such a way that when both firms make the R&D investment the total market value is increased by a factor of $5/4$.

Simultaneous symmetric R&D competition

- If both firms invest in R&D at time $t = 0$, then each will obtain an option worth 33.5, leading to an outcome of $(3.5, 3.5)$.
- If one firm alone makes the R&D investment at time $t = 0$ it preempts the competitor and captures the whole market, therefore acquiring an option worth 45, leading to final pay-offs of the form $(15, 0)$ or $(0, 15)$.
- If both firms defer the R&D investment until time $t = 1$, then they will play a game starting at that node, for which the outcomes are

		B	
		R&D at $t = 1$	No R&D at all
A	R&D at $t = 1$	**(17,17)**	(28,0)
	No R&D at all	(0,28)	(0,0)

Simultaneous symmetric R&D competition (continued)

- Therefore, for the game starting at node $t = 0$, the matrix of outcomes is

		B	
		R&D at $t = 0$	Defer R&D
A	R&D at $t = 0$	**(3.5,3.5)**	(15,0)
	Defer R&D	(0,15)	(17,17)

- Therefore the NE for the entire game consists of simultaneous investment at time $t = 0$, with an equilibrium pay-off equal to $(3.5, 3.5)$.
- Observe the PD nature of this result, therefore not optimal for the whole industry.

First mover advantage in R&D

- Let us modify the previous problem and suppose that if firm A makes an R&D investment followed by an investment firm B at time $t = 0$, then it A captures $2/3$ of the market, with $1/3$ going to B, and the market increases by $5/4$.
- On the other hand, if either firm makes an R&D investment at $t = 0$ which is not followed by the other firm in the same time period, then the former gets the entire market (at its original value).
- Finally, if both firms defer investment until time $t = 1$, then the same type of game can be played again at this node.
- To begin with, we can see that if both firms invest at time $t = 0$ (A followed by B), they acquire options which are worth 52 for firm A and 15 for firm B, leading to an outcome $(22, -15)$.
- On the other hand, if only one of the firms makes the R&D investment at time $t = 0$, they achieve the same outcomes as before, namely $(15, 0)$ and $(0, 15)$.

First-mover advantage in R&D (continued)

- Finally, if both firms defer investment until time $t = 1$, then we have a game starting at that node with the following outcomes:

		B	
		R&D at $t = 1$	No R&D at all
A	R&D at $t = 1$	**(30,3)**	(28,0)
	No R&D at all	(0,28)	(0,0)

- Using Zermelo's algorithm (sequential form on a tree), we find that the solution for this game is that A makes an investment in R&D at time $t = 0$ which is then not followed B, leading to an outcome $(15, 0)$
- Therefore, A will make full use of its first–mover advantage and preempts the competition.

Fading first–mover advantage

- Suppose now that the first–mover advantage for A disappears after time $t = 0$.
- Then the outcome for both firms investing in R&D at time zero remains $(22, -15)$, as do the outcomes $(15, 0)$ and $(0, 15)$ for investment by one firm alone at time $t = 0$.
- The novelty now is that in case both firms defer R&D until time $t = 1$, we are back at the situation of a simultaneous symmetric game played at this node, for which we know that the equilibrium outcome is $(17, 17)$.
- Using Zermelo's algorithm, the solution for the entire game now is that both firms defer R&D until time $t = 1$ and realize the outcome $(17, 17)$, which is indeed better than before for both firms.

Joint R&D venture

- If we assume that the firms can collaborate on R&D either at time $t = 0$ or at time $t = 1$, while still having the possibility to embark in R&D alone, then the outcomes for the game starting at time $t = 0$ are now

		B	
		R&D at $t = 0$	Defer R&D
A	R&D at $t = 0$	18.5,18.5	(15,0)
	Defer R&D	(0,15)	(22.5,22.5)

- We are therefore led to a situation with two NE, with the preferred outcome $(22.5, 22.5)$ being one of the possibilities.