Business V703 Financial Modeling Valuation

Lecture 10 - Examples of games in business

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Symmetric Innovation Race - SIR (Smit/Trigeorgis 04)

- Consider an innovation race for a new electronic technology between firms A and B.
- Suppose that the total net present value from immediate investment is \$400 million.
- If both firms invest, we assume that they share this value equally, whereas if only one firm invests immediately, it receives the total market value, while the other receives nothing.
- Assuming that there is uncertainty in future demand, let us say that we calculate that the value of option to invest in such market is \$600 million.
- According to our previous rules, since this is larger than the NPV, a monopolistic investor would wait, therefore owning an option worth \$600 million.
- Therefore, if both firms wait, they each own an option worth \$300 million.

Solution of the SIR game

- This symmetric innovation race can therefore be summarize as

		B: Invest	B: Wait
A	Invest	**(200,200)**	(400,0)
	Wait	(0,400)	(300,300)

- This is the business analogue of the Prisoner's dilemma, since the second row and second column are strictly dominated respectively by the first row and first column.
- Therefore, the only NE is (Invest,Invest) !
- As with the PD, an analysis of this game in extensive–form, regardless of the order the players move (or even using information sets for simultaneous moves), would lead to exactly the same solution.

Asymmetric Innovation Race - AIR (Smit/Trigeorgis 04)

- Let us introduce asymmetry in the previous problem by assuming that A has a better technological basis (say clever engineers), but somewhat restricted resources (say costlier loans).
- Suppose that if both firms jump into a high-cost development, they both end up with a negative payoff of \$100 millions (for example, because A ends up with a Pyrrhic victory).
- Next, suppose that if A leads the race (high cost) while B follows (low cost), then A receives a pay-off of \$100 and B only captures \$10 million.
- On the other hand, if B leads (high-cost) and A follows (low-cost), then B receives \$200 million, while A gets only \$10 million (recall that the cost of capital is lower for B).
- Finally, if both firms decide for a low-cost strategy, then A gets \$200 million while B ends up with \$100 million (recall that A has better engineers).

Solution for the AIR in strategic form

- This game can now be expressed in strategic form as follows:

		B: High cost	B: Low cost
A	High cost	(-100,-100)	(100,10)
	Low cost	**(10,200)**	(200,100)

- This is the business analogue for an asymmetric "grab–the–dollar" game.
- In this case, the first row is strictly dominated by the second, so A must choose a low-cost strategy. Knowing this, B then opts for a high-cost strategy.
- In other words, (Low cost, High cost) form the unique NE in this problem, with a payoff $(10, 200)$.
- This assumes, however, that the decisions need to be taken simultaneously by the two firms.

Solution for the AIR in extensive form

- The asymmetric innovation race presented in the previous page is an example where changing the order in which the players move alters the solution.
- If B gets to play first, then A will decide for a low–cost strategy regardless of what B does (recall that the first row is strictly dominated), leading to the same solution as before.
- Suppose now that A gets to play first. Since B has no dominant strategy (neither column dominates the other), the decision taken by A matters.
- If A goes "High", then B should go "Low", leading to a payoff $(100, 10)$.
- If A goes "Low", then B should go "High", leading to a payoff $(10, 200)$.
- Knowing this, A chooses the first option and the solution is (High cost,Low cost), which is the business analogue of a burning the bridges strategy.
- Let us try this example on a tree !

Asymmetric Innovation Race with Incomplete Information

- Consider the AIR game with simultaneous decision, but suppose that firm B is not completely sure about the technological capabilities of firm A.
- That is, suppose that if firm A has "normal" capabilities, than the payoffs for the game are the same as before, that is

		B	
		High cost	Low cost
A	High cost	(-100,-100)	(100,10)
	Low cost	(10,200)	(200,100)

- On the other hand, if A has "excellent" capabilities, than the payoffs change to

		B	
		High cost	Low cost
A	High cost	(50,-100)	(300,10)
	Low cost	(10,200)	(200,100)

Solution for AIR with incomplete information

- We therefore find that if A is "normal", then the solution is the same as before, that is, (Low cost,High cost), leading to a payoff $(10, 200)$.
- However, if A is "excellent", then the second row is dominated (A should prefer high cost) and B must choose a low–cost strategy.
- The only way for B to decide which strategy to follow is by assigning probabilities for the capabilities of A.
- For example, if $p = 0.6$ is the probability of A being "normal", then the expected payoff for B in a low–cost strategy is

  $$0.6 \times 100 + 0.4 \times 10 = 64,$$

  whereas the in a high–cost strategy B has an expected payoff equal to

  $$0.6 \times 200 + 0.4 \times (-100) = 80.$$

- Therefore B should choose a high–cost strategy.
- Such solution is then called a Bayesian equilibrium.

Quantity competition

- Consider the classical problem of quantity competition in a market of limited demand.
- Suppose that firms A and B need to choose production levels Q_A and Q_B.
- Assume further that the equilibrium price (determined by supply and demand) is given by the simplified relation

$$P(Q_A, Q_B) = P_0 - (Q_A + Q_B)$$

- Finally, let the cost for each firm be given by

$$\begin{aligned} C_A(Q_A) &= c_A Q_A \\ C_B(Q_B) &= c_B Q_B \end{aligned}$$

- We then have that the profits for the two firm are given by

$$\begin{aligned} \pi_A &= P(Q_A, Q_B) - C_A(Q_A) = [P_0 - c_A - (Q_A + Q_B)]Q_A \\ \pi_B &= P(Q_A, Q_B) - C_B(Q_B) = [P_0 - c_B - (Q_A + Q_B)]Q_B \end{aligned}$$

Profit table

- Taking $P_0 = 17.5$ and $c_A = c_B = 5$, the profits for each firm are given by the following table:

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	11.5	10.5	9.5	8.5	7.5	6.5	5.5	4.5	3.5	2.5
2	21	19	17	15	13	11	9	7	5	3
3	28.5	25.5	22.5	19.5	16.5	13.5	10.5	7.5	4.5	1.5
4	34	30	26	22	18	14	10	6	2	-2
5	37.5	32.5	27.5	22.5	17.5	12.5	7.5	2.5	-2.5	-7.5
6	39	33	27	21	15	9	3	-3	-9	-15
7	38.5	31.5	24.5	17.5	10.5	3.5	-3.5	-10.5	-17.5	-24.5
8	36	28	20	12	4	-4	-12	-20	-28	-36
9	31.5	22.5	13.5	4.5	-4.5	-13.5	-22.5	-31.5	-40.5	-49.5

- The different rows of this table represent the quantity level chosen by the firm whose profit we want to calculate, while the columns represent the quantity level chosen by the rival firm.
- For example, if firm A decides to produce 6 units and firm B decides to produce 3, then the profit for firm A will be 21 (row 6, column 3) while the profit for firm B will be 10.5 (row 3, column 6).

Solution to quantity competition game

- Again, we look for strictly dominated strategies and NE.
- However, due to the large number of rows and columns, it is easier to use reaction curves.
- That is, we plot the best response for A for each quantity level chosen by B and vice-versa.
- The point where this downward–sloping reaction curves intersect is then called a Nash–Cournot equilibrium, in honor to August Cournot, who analyzed this type of competition in 1838, before game theory was invented.
- For the data in this problem, we find such equilibrium to be $Q_A = Q_B = 4$, corresponding to an equilibrium price of $P = 9.5$ and profits $\pi_A = \pi_B = 18$.
- Notice that if the firms were willing to cooperate, then each would produce three units, raising their profits to 19.5 each.

Stackelberg Leadership

- The sequential version of the quantity competition game presented in the previous pages was analyzed by Heinrich von Stackelberg in 1934.
- Suppose that firm A can decide on its level of production first.
- Then for each choice made by A, firm B will try to maximize its profit according to its own reaction curve, resulting in the following profits

Q_A	0	1	2	3	4	5	6	7	8	9
Q_B	6	6	5	5	4	4	3	3	2	2
π_A	0	5.5	11	13.5	18	17.5	21	17.5	20	13.5
π_B	39	33	27.5	22.5	18	14	10.5	7.5	5	3

- Therefore, A will pick $Q_A = 6$, leading to an equilibrium price $P = 8.5$ and profits $\pi_A = 21$ and $\pi_B = 10.5$.
- This is called a Stackelberg equilibrium.
- Try to do this on a decision tree !

Price competition

- Suppose now that two firms face a price competition, in which demand is determined by the prices chosen by them.
- Assume that the quantities sold by each firm are given by

$$
\begin{aligned}
Q_A(P_A, P_B) &= Q_0 - bP_A + dP_B \\
Q_B(P_A, P_B) &= Q_0 - bP_B + dP_A
\end{aligned}
$$

- As before, each firm will have a reaction curve for the price chosen by the other firm.
- The difference now is that the curves are upward-sloping.
- The equilibrium price, called a Bertrand equilibrium, after Joseph Bertrand (1883), is the point where the two reaction curves intersect.

Types of strategic response

- As we have seen, the strategic response in quantity war consists in doing the opposite of what the rival does.
- Such reaction is called strategic substitute.
- On the other hand, in a price war the strategic response should be to replicate what the rival does.
- This is called strategic complement.
- The nature of the strategic response depends on the type of market. If capacity can be easily changed, price wars are likely to take place, whereas a quantity war is more likely to appear in industries where investment in capacity are less flexible.
- Understanding the nature of the strategic response is an essential pre-requisite for the multi-stage games to be considered in the next lectures.