Business V703 Financial Modeling

Lecture 1 - Introduction to Strategic Valuation

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Strategic decision making

We are interested in assigning monetary values to strategic decisions. Traditional, these include the decision to:

- create a new firm;
- invest in a new project;
- start a real estate development;
- finance R&D;
- abandon a non-profitable project;
- temporarily suspend operations under adverse conditions.

As we will see, many other non-financial decisions can be addressed in the same framework. For instance, the decision to:

- enroll in an MBA program;
- get married;
- change the constitution of a country;
- introduce environmental laws;
- develop a controversial highway;
- commit suicide !

Elements of Valuation

In all of the previous problems, we can identify the following common elements:

- uncertainty about the future;
- some degree of irreversibility;
- timing flexibility;
- interaction with other people's decisions.

To account for these elements, we are going to base our decisions on values obtained using the following theoretical tools:

- Net Present Value
- Real Options
- Game Theory

Net Present Value

- Takes into account the intrinsic advantages of a given investment when compared to capital markets.
- These are essentially due to market imperfections, such as entry barriers, product differentiation, economy of scale, etc...
- If we denote the expected present value of future cash flows by V_0 and the sunk cost of an immediate investment by I_0, then the Net Present Value of an investment is

$$\mathrm{NPV} = V_0 - I_0.$$

- The decision rule according to NPV is to invest whenever

$$\mathrm{NPV} > 0 \Leftrightarrow V_0 > I_0.$$

Real Options

- Takes into account the value of waiting, that is, the advantages resulting from the fact that a decision can be delayed until more favorable conditions arise.
- Uses analogies with options in financial markets to determined a value C_0 for the option to invest.
- This leads to an Extended Net Present Value for an investment given by

$$\text{ENPV} = (V_0 - I_0) - C_0. \tag{1}$$

- Accordingly, the new rule is to invest whenever

$$\text{ENPV} > 0 \Leftrightarrow V_0 - I_0 > C_0.$$

Game Theory

- Takes into account the effect of competition.
- The goal is to assign a strategic value G_0 to both conditional and unconditional moves toward investment that can create a competitive advantaged in the market.
- This leads to a Strategic Extended Net Present Value for an investment given by

$$\text{SENPV} = (V_0 - I_0) - C_0 + G_0. \tag{2}$$

- Therefore, the decision rule now is to invest whenever

$$\text{SENPV} > 0 \Leftrightarrow (V_0 - I_0) + G_0 > C_0.$$

Mathematical tools

From the previous discussion, we see that our goal is to calculate V_0, C_0 and G_0.

- V_0 can be calculated using simple probability theory (random variables, expectations, etc...).
- C_0 involves additional principles from financial mathematics (arbitrage, replication, contingent claims valuation).
- G_0 requires additional concepts from game theory (strategies, equilibrium, sequential solutions, etc...).

We will spend the first half of the course on the first two items of this list. Let us now motivate them with a simple example.

A simple example of decision under uncertainty

- Consider the decision to build a factory which produces MP3 players.
- Assume that the factory can be built instantly at cost $I_0 = \$1600$, producing one MP3 per year at zero operating cost, and that this investment is irreversible.
- Suppose that the current price for the MP3 player is $P_0 = \$200$ and that our assessment of the market is that the price next year can rise to $P_u = \$300$ with probability $q = 0.5$ or fall to $P_d = \$100$ with probability $1 - q = 0.5$, and then remain at this level forever.
- Finally, assume that the risk over these future prices is fully diversified, so that the future cash flows should be discounted using a risk–free simply compounded interest rate, which we take to be $R = 0.1$.

Net Present Value Result

- Suppose that the investment in the factory is a "now or never" decision.
- Under the conditions stated above, the present value of all future cash flows from sales of MP3 players is given by

$$V_0 = P_0 + \sum_{t=1}^{\infty} \frac{qP_u + (1-q)P_d}{(1+R)^t} = P_0 + \frac{qP_u + (1-q)P_d}{R}. \tag{3}$$

- From the values specified above, we find

$$V_0 = 200 + \frac{200}{0.1} = 2200.$$

- Since $I_0 = 1600$, the net present value for this investment is

$$\text{NPV} = V_0 - I_0 = 600 > 0,$$

from which we conclude that we should invest at $t = 0$.

The opportunity to wait

- Suppose now that our investment decision can be made now or postponed until time $t = 1$.
- Clearly, if we wait until $t = 1$, we will have the opportunity to observe the price movement and then decide to invest if it goes up or not invest if it goes down.
- In doing so, we can gain by avoiding a bad investment if the price goes down. The price we pay is that we forgo the profit from selling the MP3 player at time zero. So is it necessarily a good idea to wait ?

Delaying the decision

- If the price goes up at time $t = 1$, then the cash–flow obtained by investing in the factory at that time would be

$$\left(300 + \frac{300}{1.1} + \frac{300}{1.1^2} + \cdots\right) - 1600 = 3300 - 1600 = 1700.$$

- This means that we would make \$1700 at time $t = 1$ if prices moved up.
- If the price goes down at time $t = 1$, then the cash–flow obtained by investing in the factory at time time would be

$$\left(100 + \frac{100}{1.1} + \frac{100}{1.1^2} + \cdots\right) - 1600 = 1100 - 1600 = -500.$$

- This means that we would make \$0 at time $t = 1$ if prices moved down (since we are not forced to invest in this case).

A real options interpretation

- Taking the expected value of the two outcomes above and discounting to time $t = 0$ leads to the conclusion that the value at time $t = 0$ for the option to wait until time $t = 1$ is

$$C_0 = \frac{0.5 \times 1700 + 0.5 \times 0}{1.1} \approx 773.$$

- In this way, the extended net present value of our investment at time zero is

$$\begin{aligned} \text{ENPV} &= (V_0 - I_0) - C_0 \\ &= (2200 - 1600) - 773 = -173 < 0. \end{aligned}$$

- Therefore, our investment rule, taking into account the real option to wait, tells us that we should postpone investment.

Option Features

Let us now see how, in this example, the value of the option to invest changes with respect to changes in the following parameters:

- Initial cost I_0.
- Initial price P_0.
- Price variability.

For this, observe that our final expression for the extended net present value of the investment is

$$\text{ENPV} = \left(P_0 + \frac{qP_u + (1-q)P_d}{R} - I_0 \right) \tag{4}$$
$$- \frac{1}{1+R} \left[q \left(\frac{P_u(1+R)}{R} - I_0 \right) + (1-q) \left(\frac{P_d(1+R)}{R} - I_0 \right)^+ \right],$$

where x^+ denotes the maximum between x and 0.

Changes in the initial cost

- The combined contribution of the initial cost to expression (4) is either
$$-I_0 + \frac{q}{1+R} I_0$$
or
$$-I_0 + \frac{1}{1+R} I_0,$$
depending on whether or not $\frac{P_d(1+R)}{R} > I_0$.
- Whichever the case, this is a negative multiple of I_0
- Therefore, the extended net present value of the investment decreases with I_0.
- In other words, higher initial costs result in less incentives to invest.

Changes in the initial price

- Suppose that we change the initial price P_0 while still agreeing that, at time $t = 1$, it moves up to $P_u = uP_0$ with probability q or down to $P_d = dP_0$ with probability $1 - q$ (in our example $u = 1.5$, $d = 0.5$ and $q = 0.5$).
- Again, we observe two contributions with opposite signs from P_0 to the extended net present value. Combining them together gives either

$$P_0 + \frac{(1-q)dP_0}{R}$$

or simply P_0, depending on whether or not $\frac{P_d(1+R)}{R} > I_0$.

- Whichever the case, this is a positive multiple of P_0.
- We then see that the extended net present value of the investment increases with the initial price.
- That is, higher initial prices result in more incentives to invest immediately.

Changes in price variability

- We can measure the effect of uncertainty in the price by arranging the parameters q, u and d in such a way that its expected value remains the same, but with a greater variance.
- For example, fix $q = 0.5$ and consider the case where $u = 1.75$ and $d = 0.25$. Then the expected value that an initial price $P_0 = 200$ is still \$200, while the spread between the upward value of \$350 and the downward values of \$50 is now much larger.
- We see that the contribution arising from terms with u and d in (4) is either zero (when d is close to 1) or $(1 - q)dP_0/R$.
- That is, the extended net present value decrease as d gets smaller.
- In other words, uncertainty discourages investment.