Assignment 2 - Math 799
Numerical Methods for Finance

04/02/2003

Consider the following stochastic differential equations:

$$
\begin{aligned}
dX_t &= \mu X_t dt + \sigma X_t dW_t & (1)\\
dX_t &= \mu X_t dt + \sigma dW_t & (2)\\
dX_t &= (\mu - X_t)dt + \sigma dW_t, & (3)
\end{aligned}
$$

where $\mu \in \mathbf{R}$ and $\sigma \geq 0$ are constants and W_t is a standard Brownian motion starting at zero (these equations define a geometric Brownian motion, an Ornstein-Uhlenbeck process and a mean-reverting Ornstein-Uhlenbeck process, respectively). For each equation:

- find the solution X_t (and use Ito's formula to show that it satisfies the given SDE);
- obtain $E[X_t]$ and $\mathrm{Var}[X_t]$;
- simulate 500 different paths of X_t, using a time horizon $T = 1$ (a year) and time increments $\delta t = 1/365$ (a day), and store the result on a 500×365 matrix;
- plot two of your simulated paths, as well as the function $E[X_t]$, on the same figure;
- for each $i = 1, \ldots, 365$, calculate the sample mean $\bar{\mu}$ of the random variable X_i over the 500 simulated paths and plot the result together with the fucntion $E[X_t]$. Repeat the procedure for the sample variance $\bar{\sigma}^2$ and the fucntion $\mathrm{Var}[X_t]$.