

Assignment 4, Math 712

Due Apr. 25, emailed to me in scanned pdf format (no pictures please)

1. Prove the Łoś Theorem for continuous logic i.e. suppose  $M_i$  are  $L$ -structures for some continuous language  $L$  for each  $i \in I$ ,  $U$  is an ultrafilter on  $I$  and  $M = \prod_U M_i$ . If  $\varphi(x_1, \dots, x_n)$  is an  $L$ -formula then

$$\varphi^M(a_1, \dots, a_n) = \lim_U \varphi^{M_i}(a_1^i, \dots, a_n^i)$$

for any  $a_1, \dots, a_n \in M$ .

2. Show that the operator norm unit ball of  $M_n(C)$  is compact in the 2-norm given by the trace i.e. basic open sets look like

$$\{A \in M_n(C) : \|A\| \leq 1 \text{ and } \|A - B\|_2 < \epsilon\}$$

for any  $\epsilon$  and any  $B \in M_n(C)$  with  $\|B\| \leq 1$ .